Learning Decision Trees

Machine Learning

THE UNIVERSITY OF UTAH

Some slides from Tom Mitchell, Dan Roth and others

This lecture: Learning Decision Trees

1. **Representation**: What are decision trees?

2. **Algorithm**: Learning decision trees
   – The ID3 algorithm: A greedy heuristic

3. Some extensions

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2. **Algorithm**: Learning decision trees
   – The ID3 algorithm: A greedy heuristic

3. Some extensions

History of Decision Tree Research

- Full search decision tree methods to model human concept learning: Hunt et al 60s, psychology
- Quinlan developed the ID3 (***I****terative* ***D****ichotomiser* ***3***) algorithm, with the information gain heuristic to learn expert systems from examples (late 70s)
- Breiman, Freidman and colleagues in statistics developed CART (**C**lassification **A**nd **R**egression **T**rees, mid 80s)
- Many improvements in the 80s: coping with noise, continuous attributes, missing data, non-axis parallel, etc.
- Quinlan's updated algorithms, C4.5 (1993) and C5 are more commonly used
- Boosting (or Bagging) over decision trees is a very good general-purpose algorithm

Will I play tennis today?

- **Features**
  - Outlook: {Sun, Overcast, Rain}
  - Temperature: {Hot, Mild, Cool}
  - Humidity: {High, Normal, Low}
  - Wind: {Strong, Weak}

- **Labels**
  - Binary classification task: Y = {+, -}

Will I play tennis today?

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

Outlook: **S**unny, **O**vercast, **R**ainy

Temperature: **H**ot, **M**edium, **C**ool

Humidity: **H**igh, **N**ormal, **L**ow

Wind: **S**trong, **W**eak

Basic Decision Tree Learning Algorithm

- Data processed as a batch (i.e. all data available)

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9	S	C	N	W	+
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12	O	M	H	S	+
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Basic Decision Tree Learning Algorithm

- Data processed as a batch (i.e. all data available)
- Recursively build a decision tree top down.

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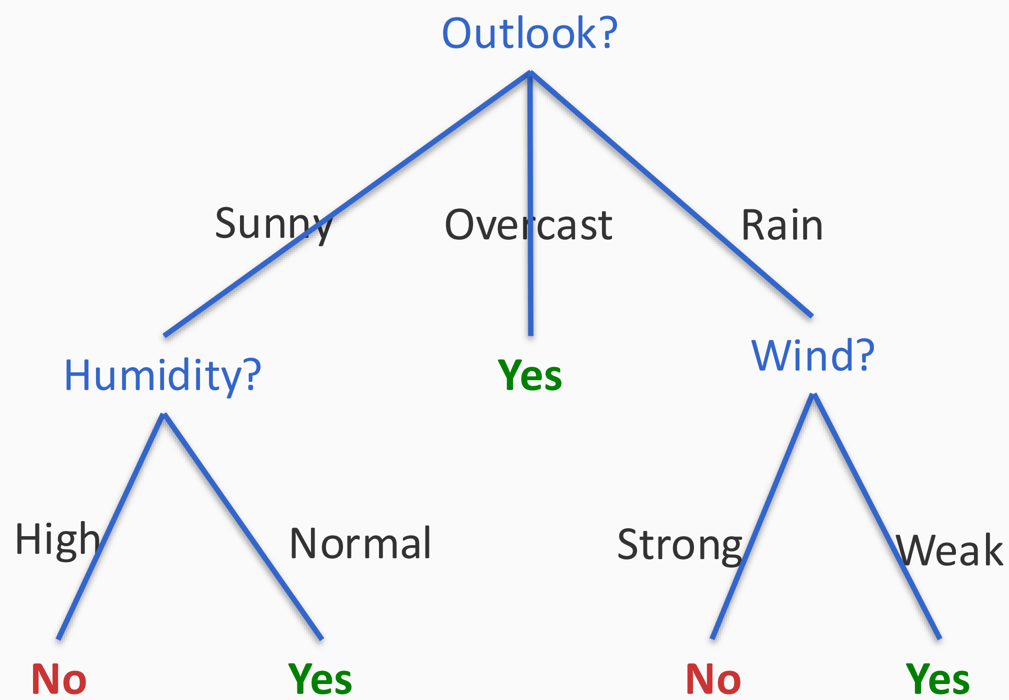

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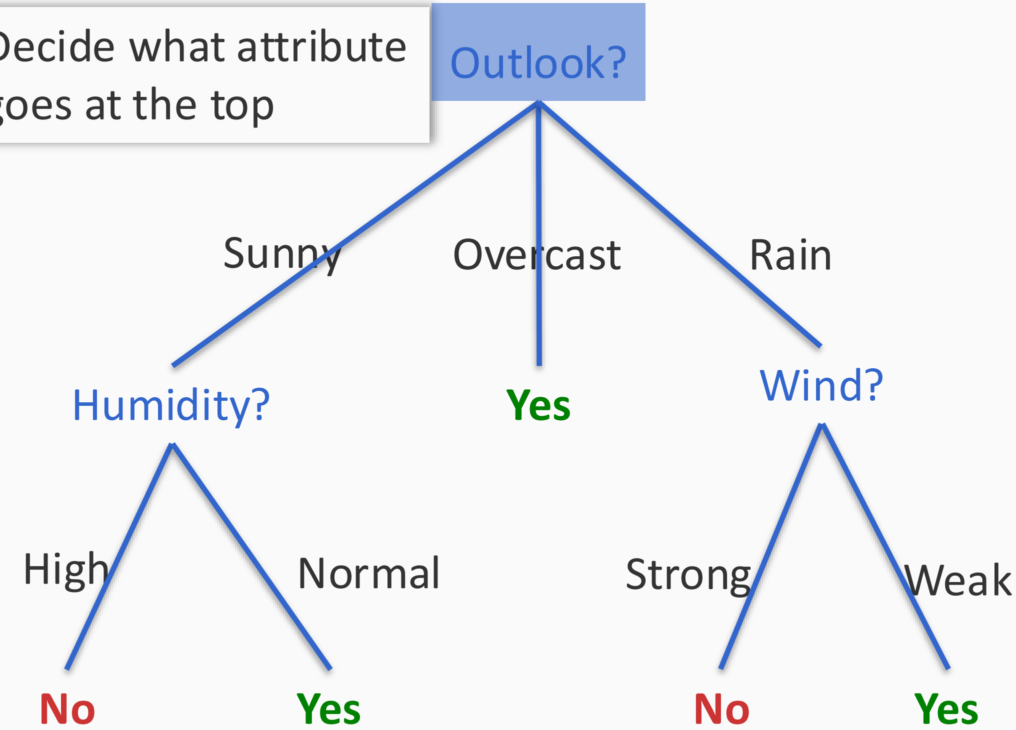
1. Decide what attribute goes at the top



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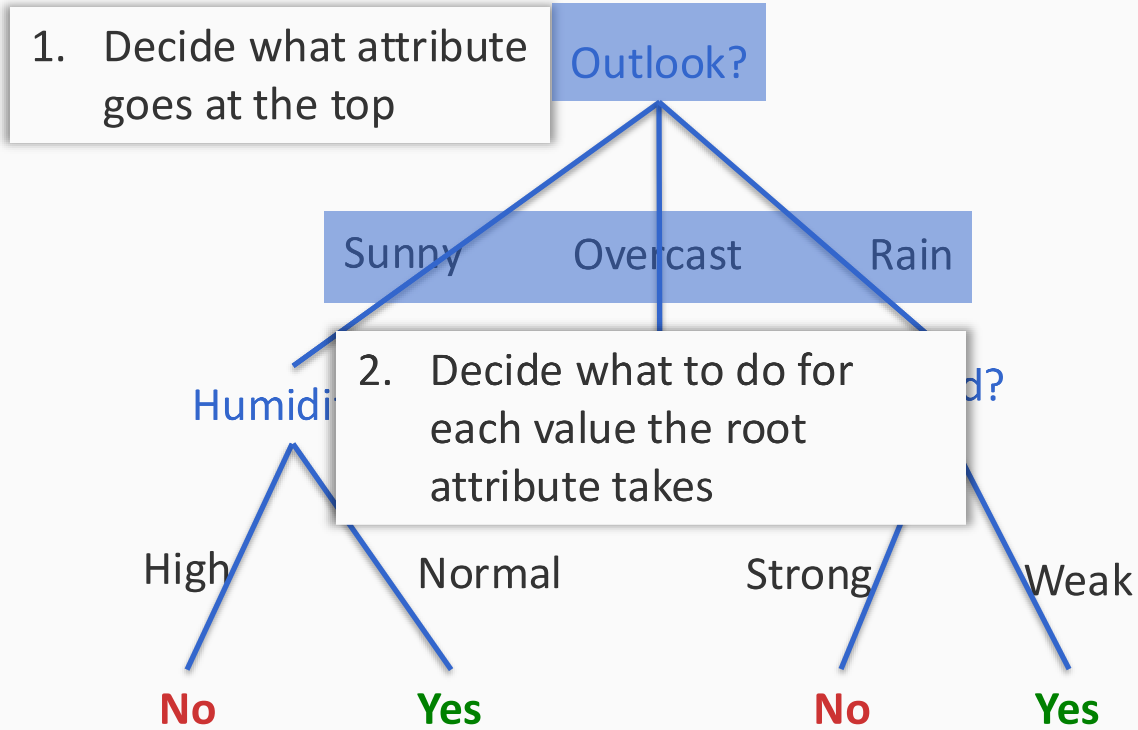

Basic Decision Tree Algorithm: ID3

ID3(S, Attributes):

Basic Decision Tree Algorithm: ID3

ID3(S, Attributes):

Dataset of labeled examples

Basic Decision Tree Algorithm: ID3

ID3(S, Attributes):

The set of attributes (i.e. features) that are measured for each example

Dataset of labeled examples

Basic Decision Tree Algorithm: ID3

Input:
S the set of Examples
Attributes is the set of measured attributes

ID3(S, Attributes):

1. If all examples are have same label:
   Return a single node tree with the label

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      Decide what attribute goes at the top

   4. Return `Root node`

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   2. A = attribute in Attributes that ***best*** classifies S

   4. Return `Root node`

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   2. A = attribute in Attributes that ***best*** classifies S
   3. for each possible value v of that A can take:

   4. Return `Root node`

Decide what to do for each value the root attribute takes

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         add leaf node with the common value of Label in S

         why?
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why?

For generalization at test time

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      2. Let S_v be the subset of examples in S with A=v
      3. if S_v is empty:
         add leaf node with the common value of Label in S
         Else:
         below this branch add the subtree ID3(S_v, Attributes - {A})
   4. Return `Root node`

Decide what to do for each value the root attribute takes

why?

For generalization at test time

Recursive call to the ID3 algorithm with all the remaining attributes

Picking the Root Attribute

- Goal: Have the resulting decision tree **as small as possible** (*Occam's Razor*)
  - But, finding the minimal decision tree consistent with data is NP-hard

- The recursive algorithm is a greedy heuristic search for a simple tree, but cannot guarantee optimality

- The main decision in the algorithm is the selection of the next attribute to split on

Picking the Root Attribute

Consider data with two Boolean attributes (A,B).

< (A=0,B=0), − >: 50 examples

< (A=0,B=1), − >: 50 examples

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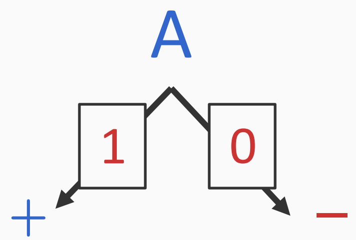
Splitting on A: we get purely labeled nodes.



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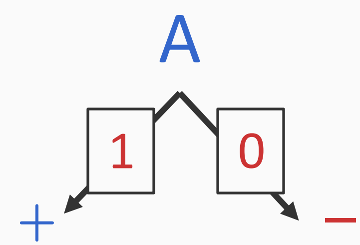

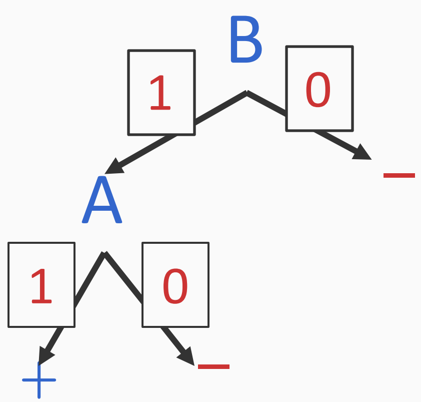

Splitting on B: we don't get purely labeled nodes.

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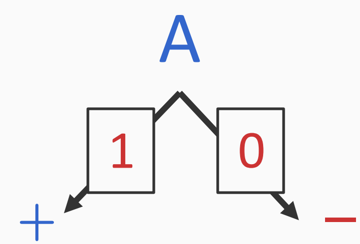

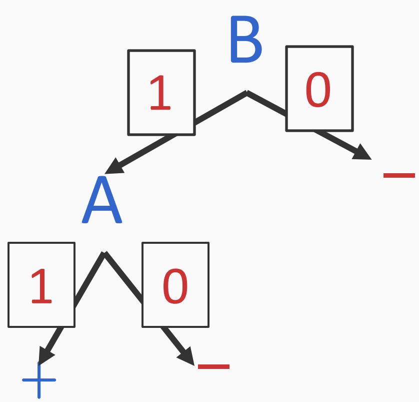

Splitting on B: we don't get purely labeled nodes.

What if we have: <(A=1,B=0), - >: 3 examples

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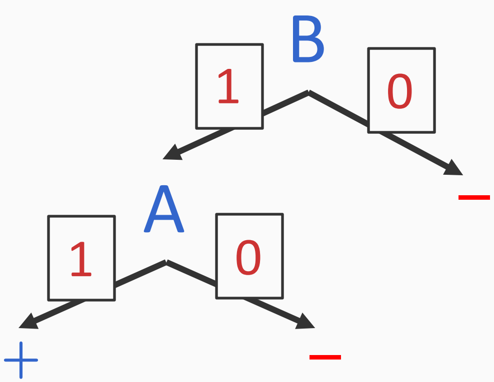

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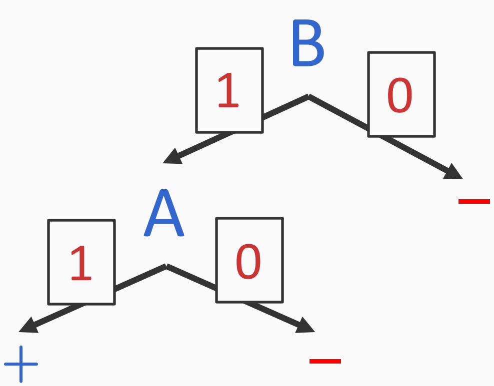

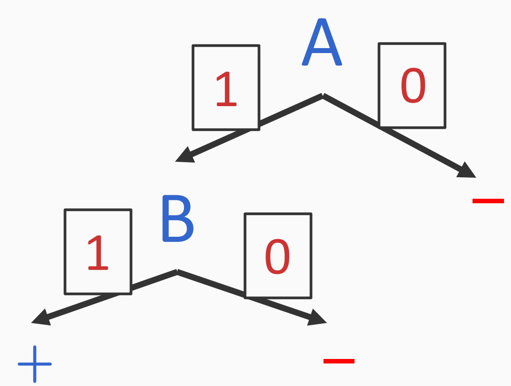

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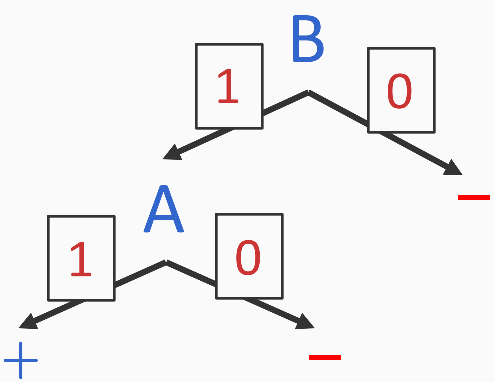
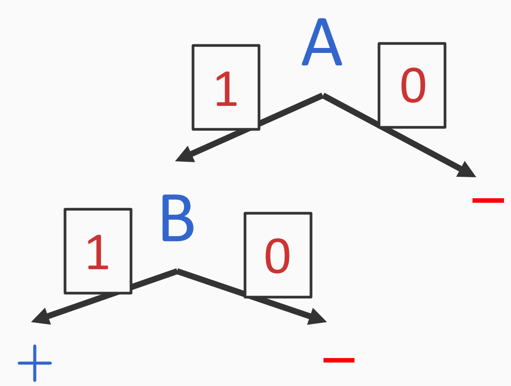
Which attribute should we choose? Trees looks structurally similar!





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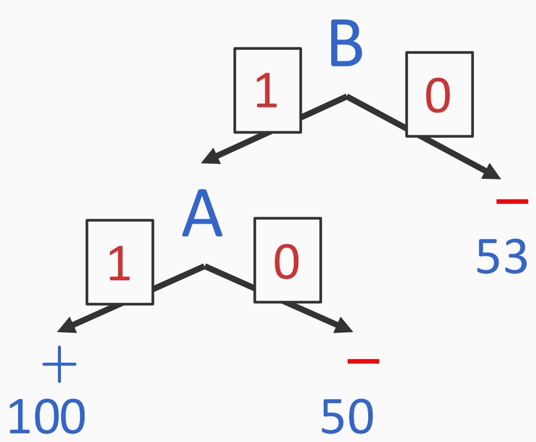

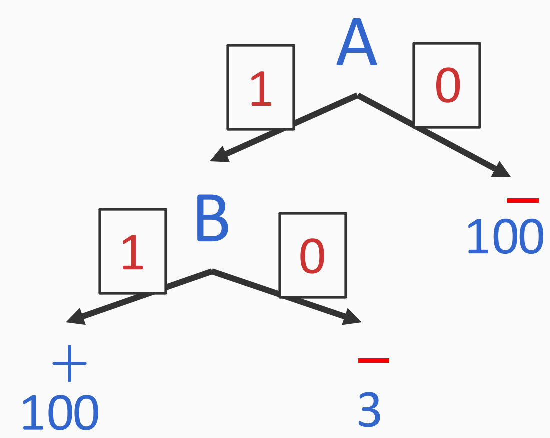

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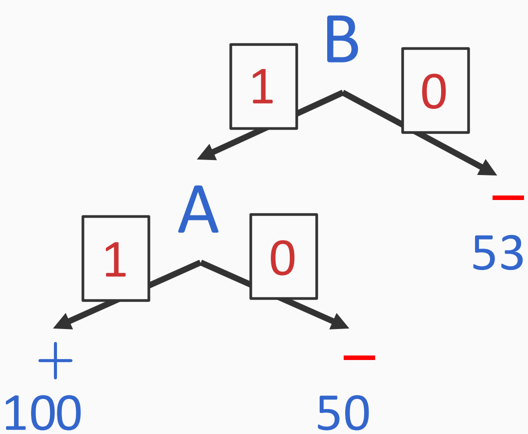
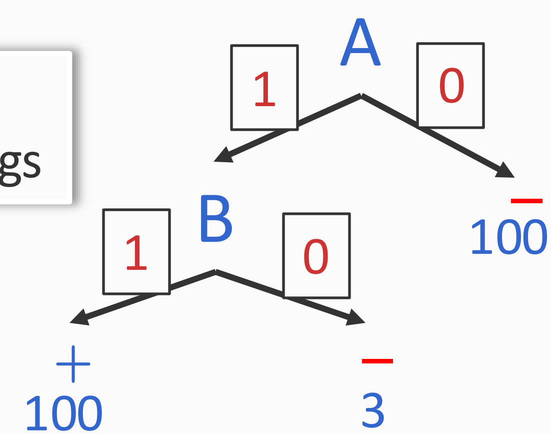


Advantage A. But…
Need a way to quantify things



Picking the Root Attribute

Goal: Have the resulting decision tree **as small as possible** (*Occam's Razor*)

- The main decision in the algorithm is the selection of the next attribute for splitting the data
- We want attributes that split the examples to sets that are relatively pure in one label
  - This way we are closer to a leaf node.
- The most popular heuristic is information gain, originated with the ID3 system of Quinlan

Reminder: Entropy

Entropy (impurity, disorder) of a set of examples S with respect to binary classification is

$$Entropy(S) = H(S) = -p_+ \log_2(p_+) - p_- \log_2(p_-)$$

- The proportion of positive examples is p_+
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In general, for a discrete probability distribution with K possible values, with probabilities $\{p_1, p_2, \cdots, p_k\}$ the entropy is given by

$$H(\{p_1, p_2, \cdots, p_k\}) = -\sum_{i}^{K} p_i \log_2 \, p_i$$

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Entropy can be viewed as the number of bits required, on average, to encode information.

If the probability for + is 0.5, a single bit is required for each example; if it is 0.8, we can use less then 1 bit.

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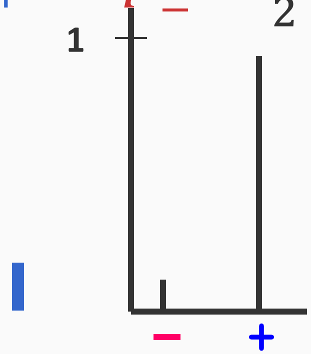

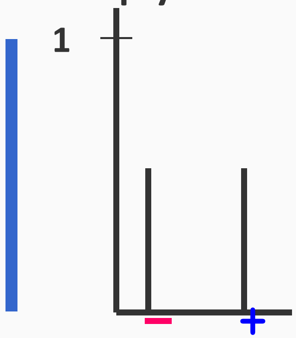

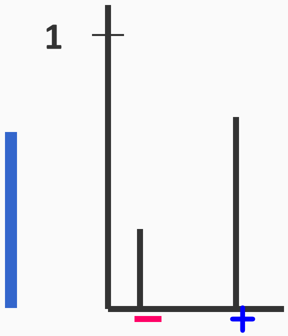

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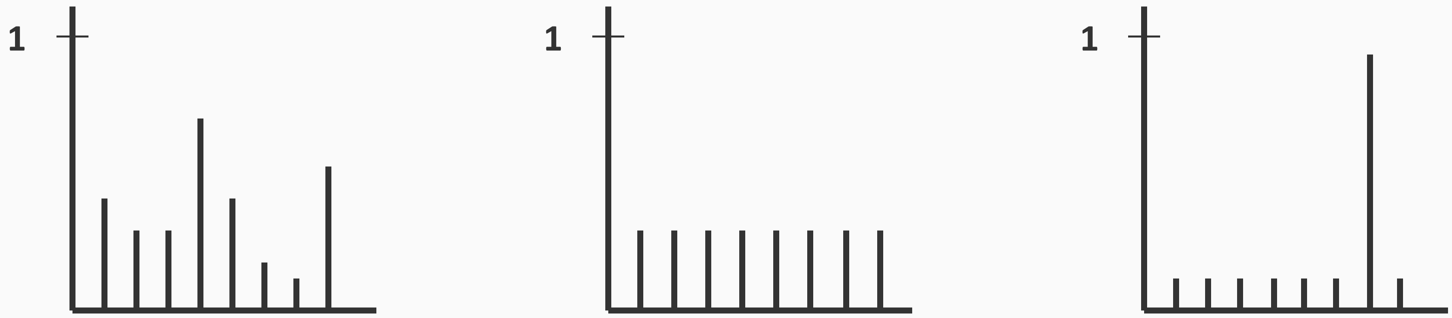
The uniform distribution has the highest entropy



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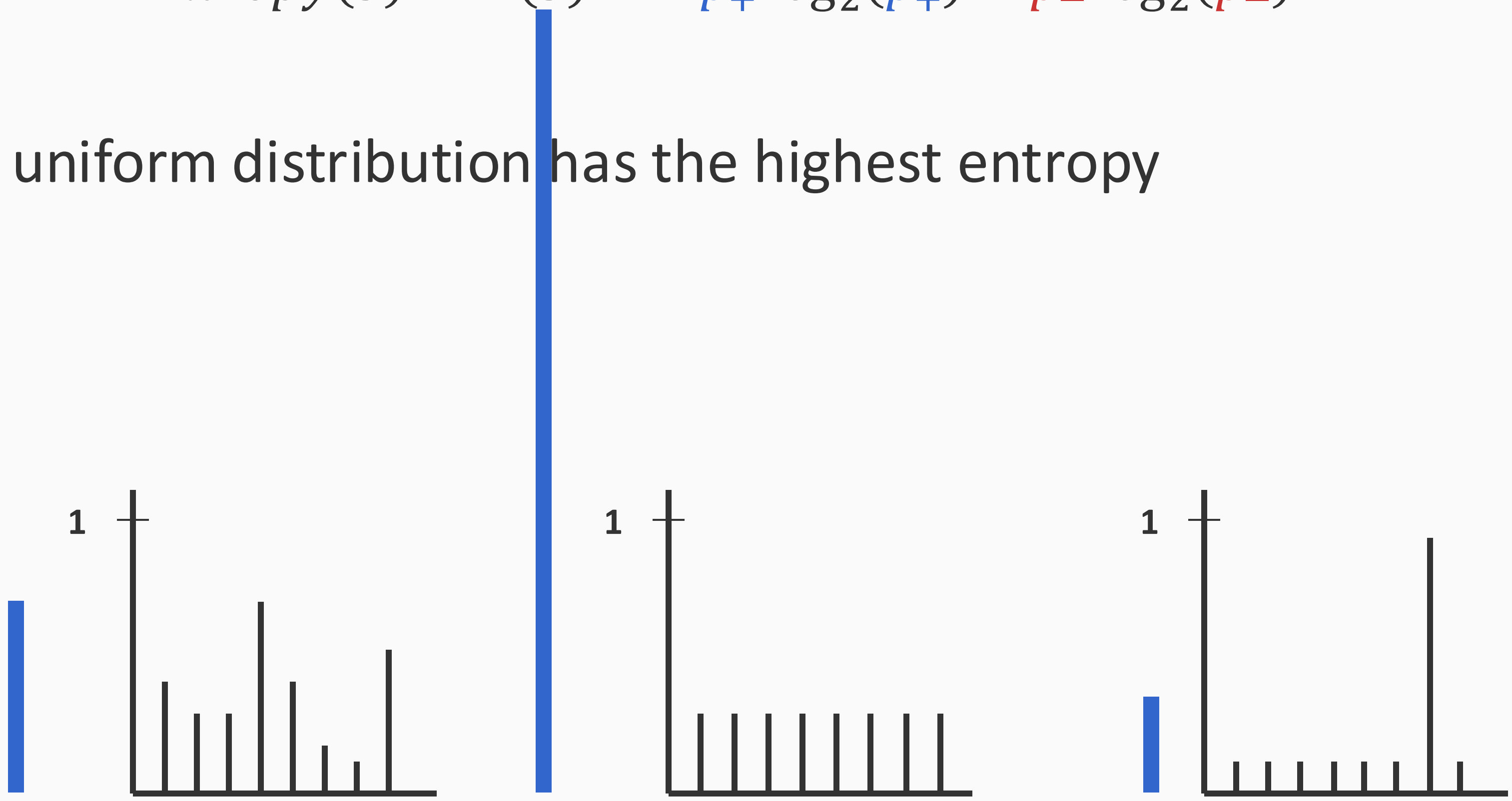

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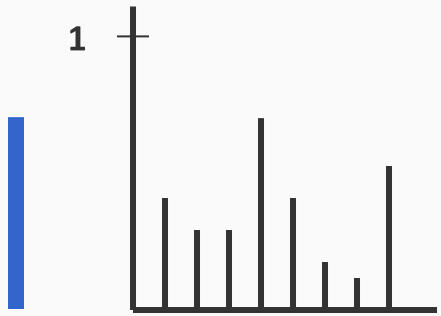
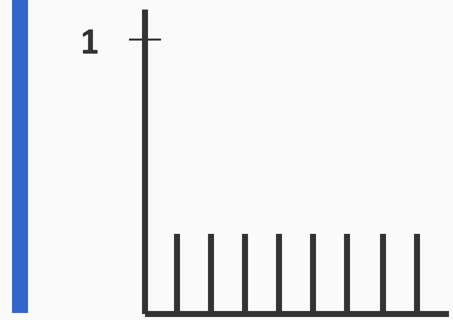
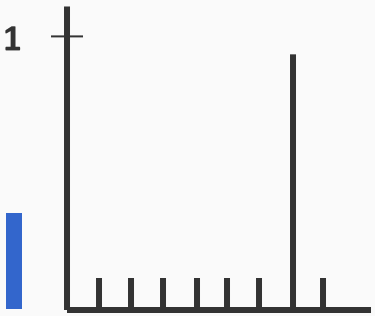
The uniform d

High Entropy: High level of Uncertainty

Low Entropy: Low Uncertainty







Picking the Root Attribute

Goal: Have the resulting decision tree **as small as possible** (*Occam's Razor*)

- The main decision in the algorithm is the selection of the next attribute for splitting the data
- We want attributes that split the examples to sets that are relatively pure in one label
  - This way we are closer to a leaf node.
- The most popular heuristic is information gain, originated with the ID3 system of Quinlan

Intuition: Choose the attribute that reduces the label entropy the most

Information Gain

The *information gain* of an attribute A is the expected reduction in entropy caused by partitioning on this attribute

$$Gain(S, A) = Entropy(S) - \sum_{v \in \text{Values}(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

S_v: the subset of examples where the value of attribute A is set to value v

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Entropy of partitioning the data is calculated by weighing the entropy of each partition by its size relative to the original set

- Partitions of low entropy (imbalanced splits) lead to high gain

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Go back to check which of the A, B splits is better

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High Entropy: High level of Uncertainty

Low Entropy: Low Uncertainty

S_v: the subs ttribute A is set to value v

Entropy of partitioning the data is calculated by weighing the entropy of each partition by its size relative to the original set

- Partitions of low entropy (imbalanced splits) lead to high gain

Go back to check which of the A, B splits is better

Will I play tennis today?

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

Outlook: S(unny), O(vercast), R(ainy)

Temperature: H(ot), M(edium), C(ool)

Humidity: H(igh), N(ormal), L(ow)

Wind: S(trong), W(eak)

Will I play tennis today?

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

Current entropy:
p = 9/14
n = 5/14

$H(\text{Play?}) = -(9/14)\log_2(9/14) - (5/14)\log_2(5/14)$
≈ 0.94

Information Gain: Outlook

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

Information Gain: Outlook

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

Outlook = sunny: 5 of 14 examples

$p = 2/5$ $n = 3/5$ $\mathbf{H_S = 0.971}$

Information Gain: Outlook

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

Outlook = sunny: 5 of 14 examples

$p = 2/5$ $n = 3/5$ $\mathbf{H_S = 0.971}$

Outlook = overcast: 4 of 14 examples

$p = 4/4$ $n = 0$ $\mathbf{H_o = 0}$

Information Gain: Outlook

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

Outlook = sunny: 5 of 14 examples

$p = 2/5$ $n = 3/5$ $\mathbf{H_S = 0.971}$

Outlook = overcast: 4 of 14 examples

$p = 4/4$ $n = 0$ $\mathbf{H_o = 0}$

Outlook = rainy: 5 of 14 examples

$p = 3/5$ $n = 2/5$ $\mathbf{H_R = 0.971}$

Expected entropy:

$(5/14)\times0.971 + (4/14)\times0 + (5/14)\times0.971$

$= \mathbf{0.694}$

Information gain:

$0.940 - 0.694 = \mathbf{0.246}$

Information Gain: Humidity

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

Information Gain: Humidity

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

Humidity = High:

$p = 3/7$ $n = 4/7$ $\mathbf{H_h = 0.985}$

Information Gain: Humidity

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

Humidity = High:

$p = 3/7$ $n = 4/7$ $\mathbf{H_h = 0.985}$

Humidity = Normal:

$p = 6/7$ $n = 1/7$ $\mathbf{H_o = 0.592}$

Expected entropy:

$(7/14)\times 0.985 + (7/14)\times 0.592 = \mathbf{0.7885}$

Information Gain: Humidity

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

Humidity = High:

$p = 3/7$ $n = 4/7$ $\mathbf{H_h = 0.985}$

Humidity = Normal:

$p = 6/7$ $n = 1/7$ $\mathbf{H_o = 0.592}$

Expected entropy:

$(7/14)\times0.985 + (7/14)\times0.592 =$ **0.7885**

Information gain:

$0.940 - 0.7885 =$ **0.1515**

Which feature to split on?

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

Information gain:

Outlook: 0.246
Humidity: 0.151
Wind: 0.048
Temperature: 0.029

Which feature to split on?

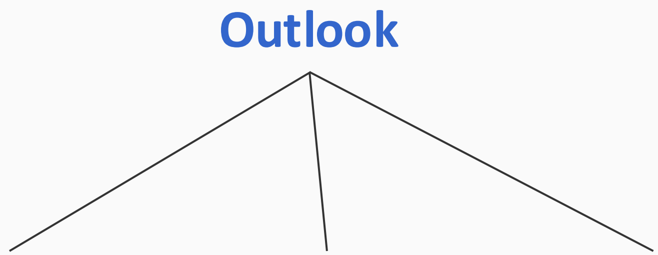
	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

Information gain:

Outlook: 0.246
Humidity: 0.151
Wind: 0.048
Temperature: 0.029

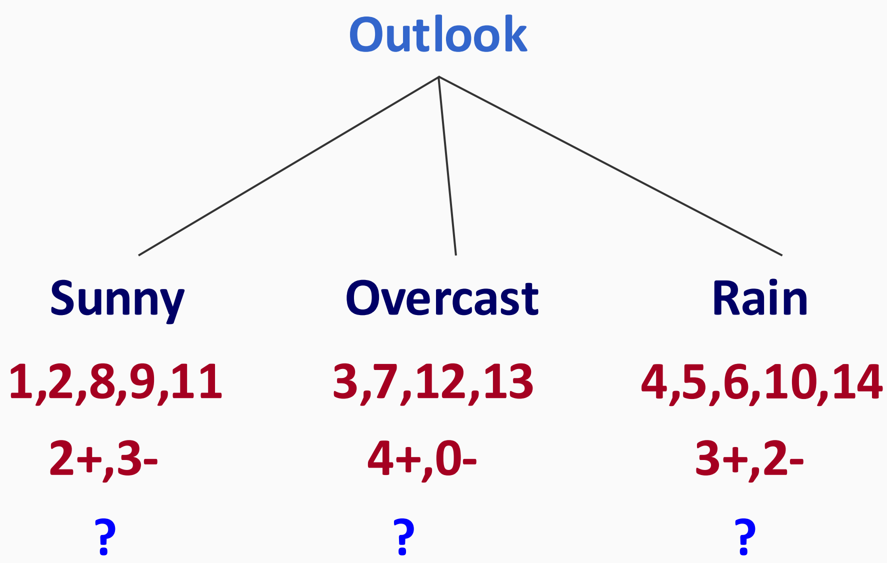
→ Split on Outlook

An Illustrative Example



Gain(S,Humidity)=0.151
Gain(S,Wind) = 0.048
Gain(S,Temperature) = 0.029
Gain(S,Outlook) = 0.246

An Illustrative Example



	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

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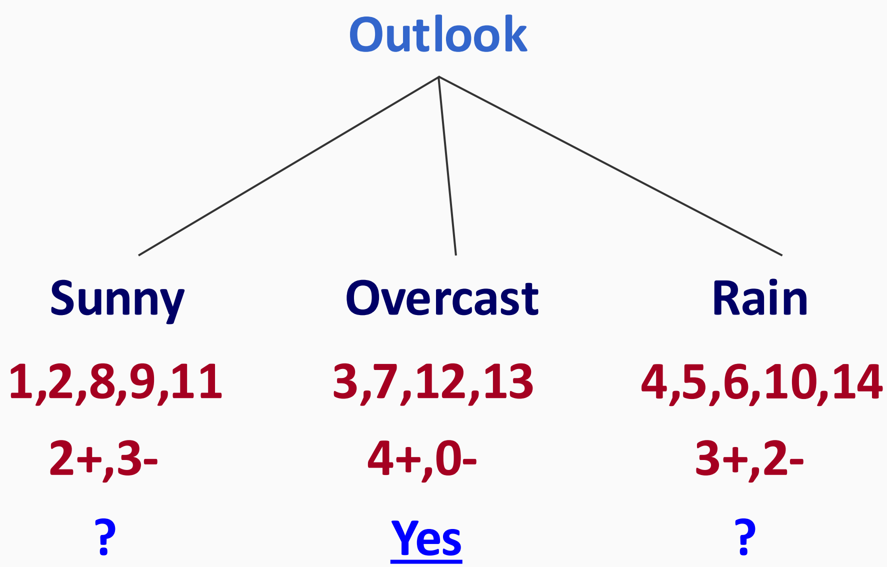

	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

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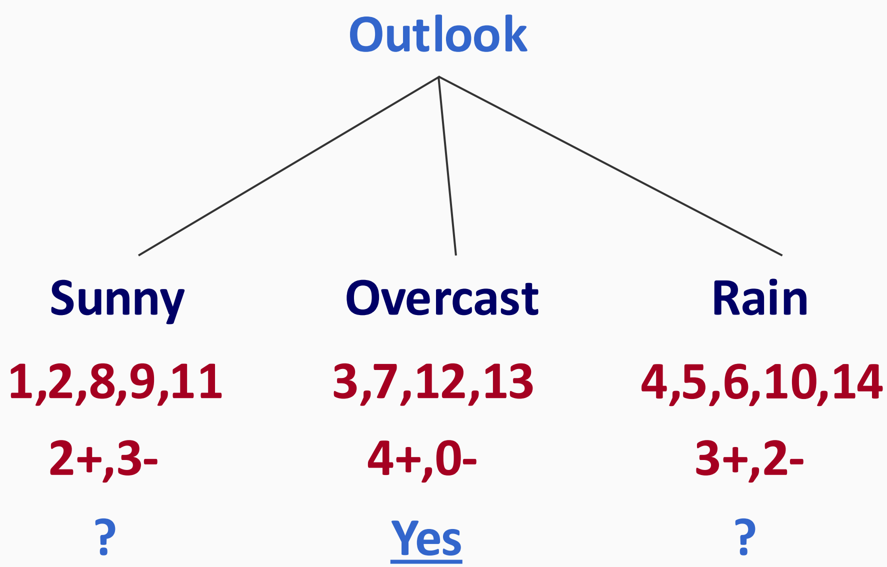
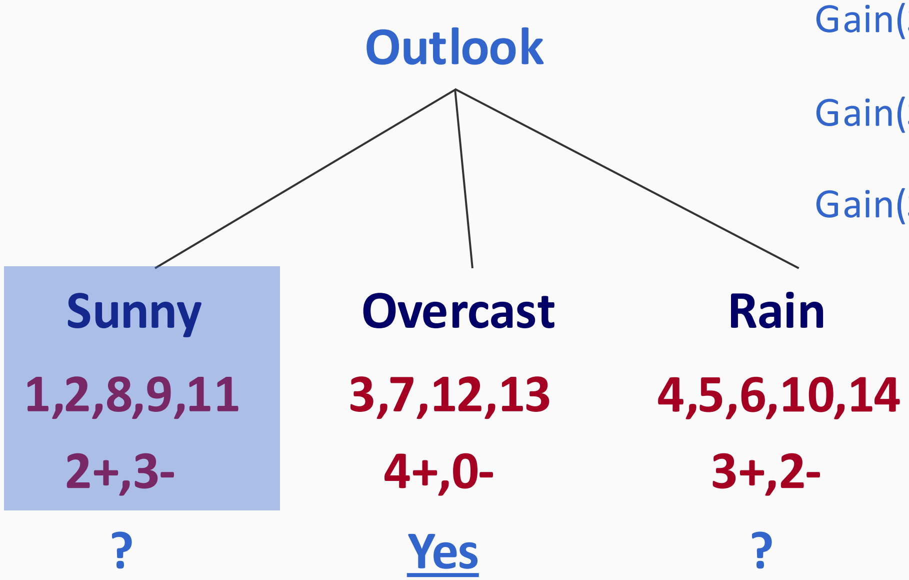


	O	T	H	W	Play?
1	S	H	H	W	-
2	S	H	H	S	-
3	O	H	H	W	+
4	R	M	H	W	+
5	R	C	N	W	+
6	R	C	N	S	-
7	O	C	N	S	+
8	S	M	H	W	-
9	S	C	N	W	+
10	R	M	N	W	+
11	S	M	N	S	+
12	O	M	H	S	+
13	O	H	N	W	+
14	R	M	H	S	-

Continue until:

- Every attribute is included in path, or,
- All examples in the leaf have same label

An Illustrative Example



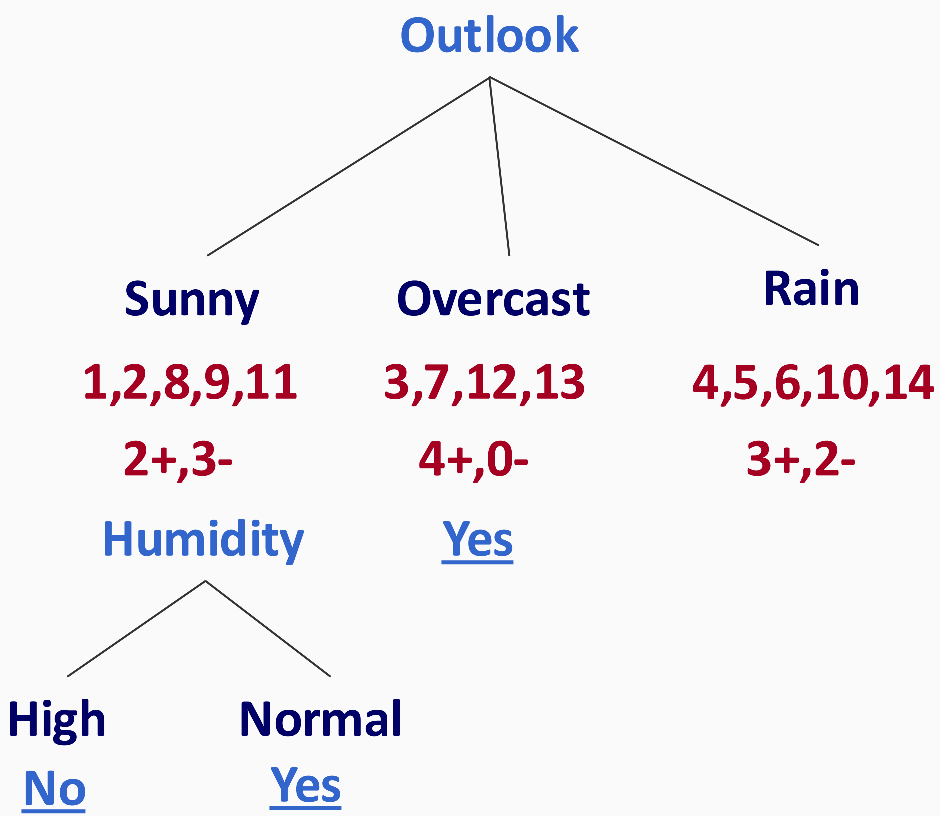
$\text{Gain}(S_{sunny}, \text{Humidity}) = .97-(3/5)\ 0-(2/5)\ 0 = .97$

$\text{Gain}(S_{sunny}, \text{Temp}) = .97-\ 0-(2/5)\ 1 = .57$

$\text{Gain}(S_{sunny}, \text{wind}) = .97-(2/5)\ 1 - (3/5)\ .92 = .02$

Day	Outlook	Temperature	Humidity	Wind	PlayTennis
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes

An Illustrative Example



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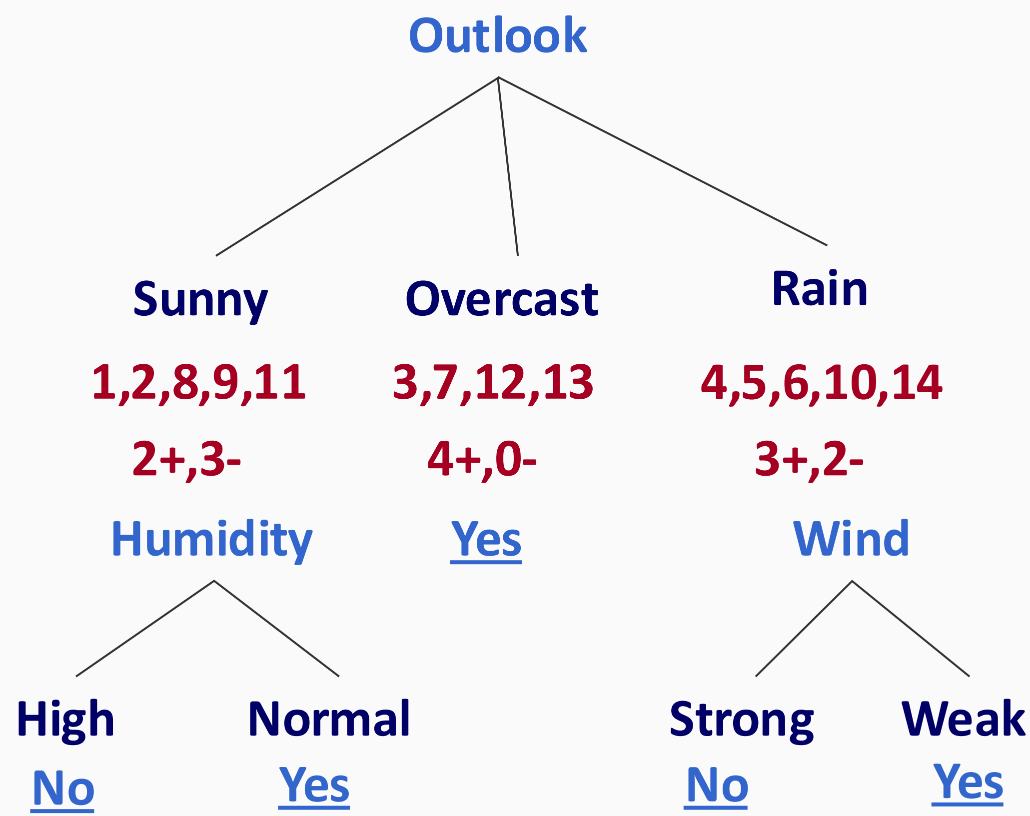

Hypothesis Space in Decision Tree Induction

- Search over decision trees, which can represent all possible discrete functions (has pros and cons)
- Goal: to find the best decision tree
- Finding a minimal decision tree consistent with a set of data is NP-hard.
- ID3 performs a greedy heuristic search
  - hill climbing without backtracking
- Makes statistical decisions using all data

Summary: Learning Decision Trees

1. **Representation**: What are decision trees?
   – A hierarchical data structure that represents data
2. **Algorithm**: Learning decision trees

   The ID3 algorithm: A greedy heuristic

   - If all the examples have the same label, create a leaf with that label
   - Otherwise, find the "most informative" attribute and split the data for different values of that attributes
   - Recurse on the splits