Learning to Search

CS 6355: Structured Prediction

Some slides adapted from Daumé and Ross

Inference

- What is inference?
  - An overview of what we have seen before
  - Combinatorial optimization
  - Different views of inference
- Graph algorithms
  - Dynamic programming, greedy algorithms, search
- Integer programming
- Heuristics for inference
  - Sampling
- Learning to search

Learning to Search

We have seen that inference as graph search

- Iteratively construct a series of partial structures
- Find the highest scoring structure in this fashion

Can we learn a model that is designed with such inference in mind?

- Learning to search is a way of formulating structured prediction problems as a search problem
- Integrates learning and prediction into a unified framework

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   - Search problems and a generic search algorithm
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3. LaSO: Learning as Search Optimization
4. SEARN: Search and Learning
5. DAgger: Dataset Aggregation

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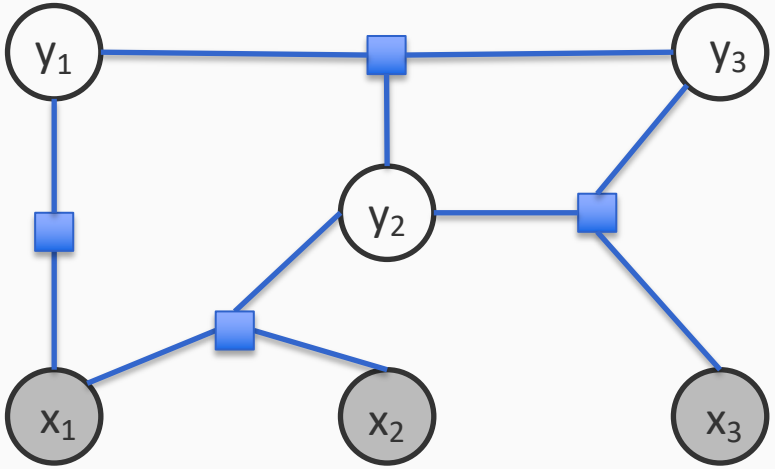

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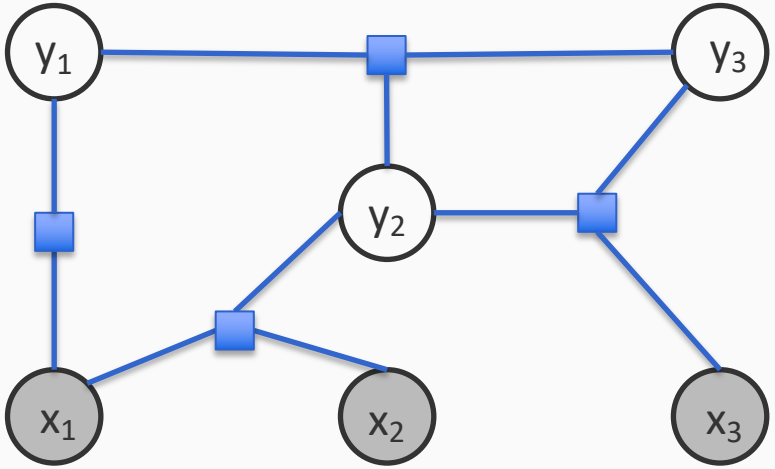

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The cost vector for this input x can be:

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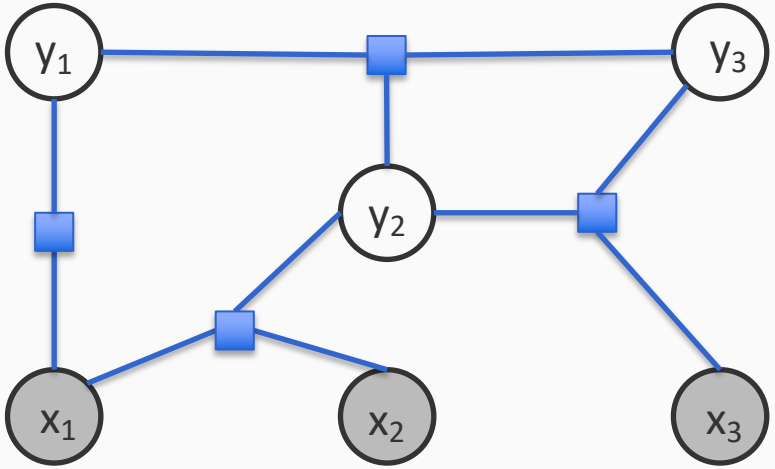

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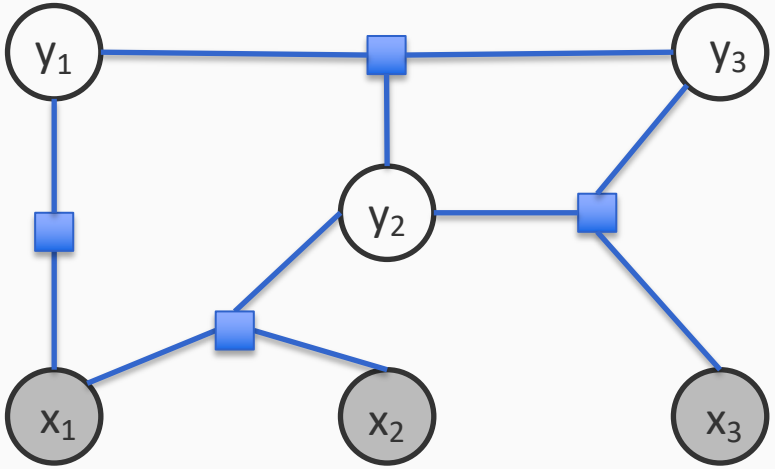
or

$$\begin{aligned} c(A,A,A) &= 2 \\ c(A,A,B) &= 2 \\ c(A,A,C) &= 1 \\ &\dots \\ c(A,B,C) &= 0 \\ &\dots \\ c(C,C,B) &= 3 \\ c(C,C,C) &= 2 \end{aligned}$$

Hamming Distance

Learning to minimize prediction cost



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Hamming Distance

What is the dimensionality of the cost vector c?

A Structured Prediction Problem

Learn a mapping $h(\mathbf{x})$ from inputs $\mathbf{x}$ to outputs $\mathbf{y}$

- Each $\mathbf{y}$ decomposes into decisions/labels $(y_1, y_2, \ldots, y_T)$
- Each $\mathbf{x}$ is associated with a cost vector $\mathbf{c}$
  - $\mathbf{c}$ has 2^{T} components if the y_i's are binary
    - Each component specifies the cost of the corresponding full assignment $\mathbf{y}$
  - Sometimes thought of as a function
- The goal is to minimize $L(h) = E[c_{h(x)}]$

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Formalizing search problems

- Initial state: denoted by s_0
  - The starting point for the search
- Actions: Actions(s)
  - The set of actions that can be performed at a state
- Transition: Result(s, a)
  - "Applies" an action a to a state s to produce the next state
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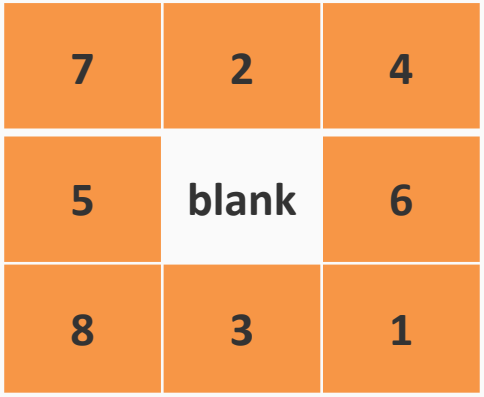
A *solution* is an action sequence that leads from initial state to a goal state.

An *optimal solution* has the lowest path cost or highest score.

Example Search Problem: 8-puzzle



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Goal State

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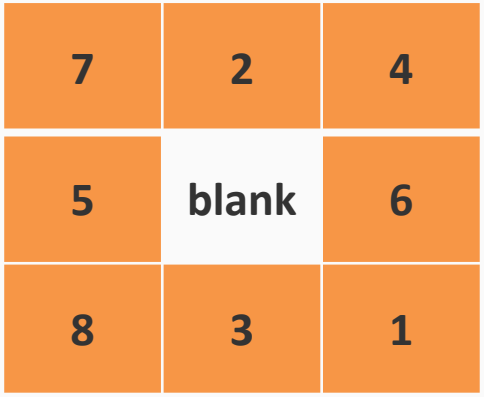

Initial State

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What are these five components for 8-puzzle?

Initial state: s_0
Actions: Actions(s)
Transition model: Result(s, a)
Goal test
Path cost / score

Generic Search Algorithm

How do we solve a search problem?

Answer: By starting at the initial state, and navigating the state space till we get to an answer

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All magic happens in the *enqueue* function (BFS, DFS, beam, A*)

Or is there any magic?

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Learning to search: General setting

Predicting an output $\mathbf{y}$ as a sequence of decisions

The high level idea:

- Frame the problem of structured prediction as a generic search problem

- Learn to enqueue nodes so that "good" states are explored first, and we get to a solution easily.

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General data structures

- State: Partial assignments to $(y_1, y_2, \ldots, y_T)$

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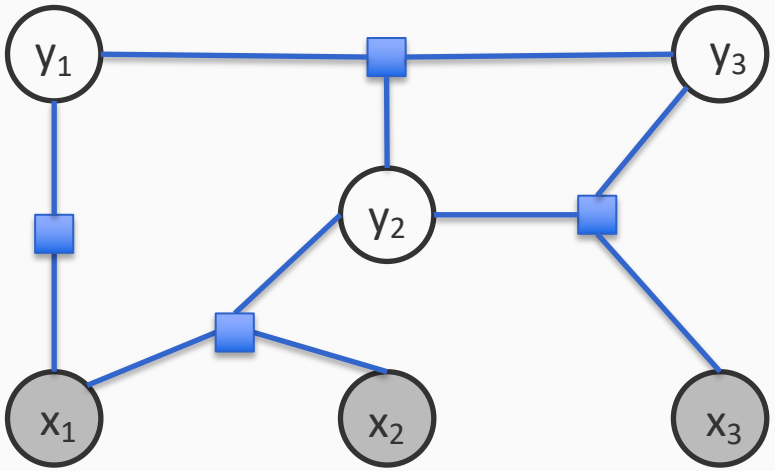
- Path cost/score function: $\mathbf{w}^T \phi(\mathbf{x}, \text{node})$
  - or, a neural network that depends on the $\mathbf{x}$ and the node
  - A node contains the current state and the back pointer to trace back the search path

Example



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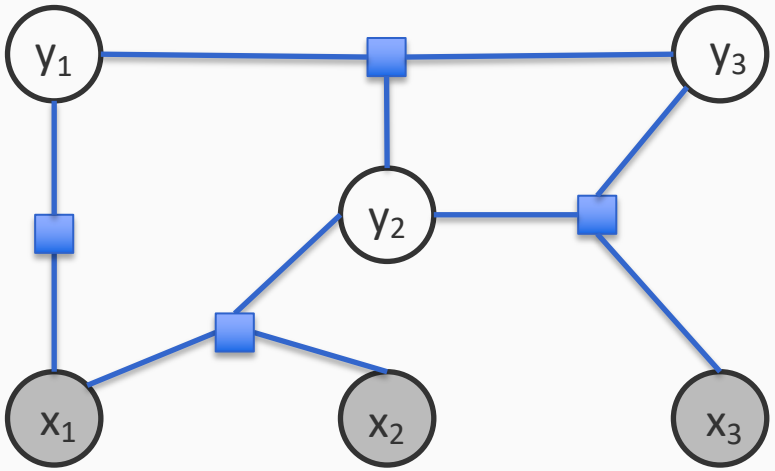

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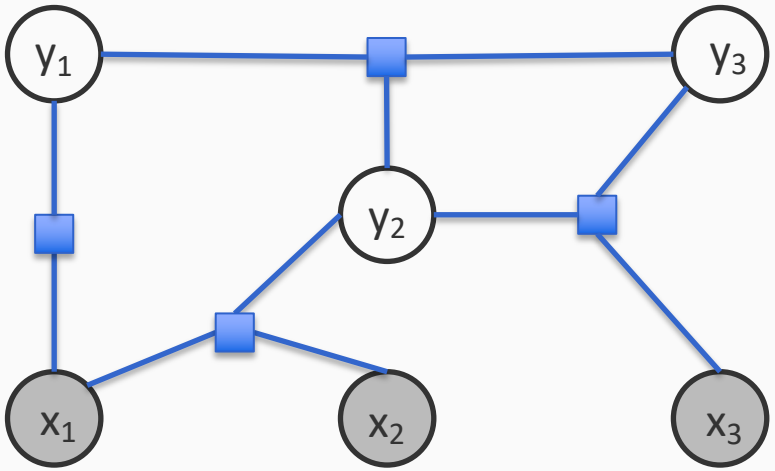

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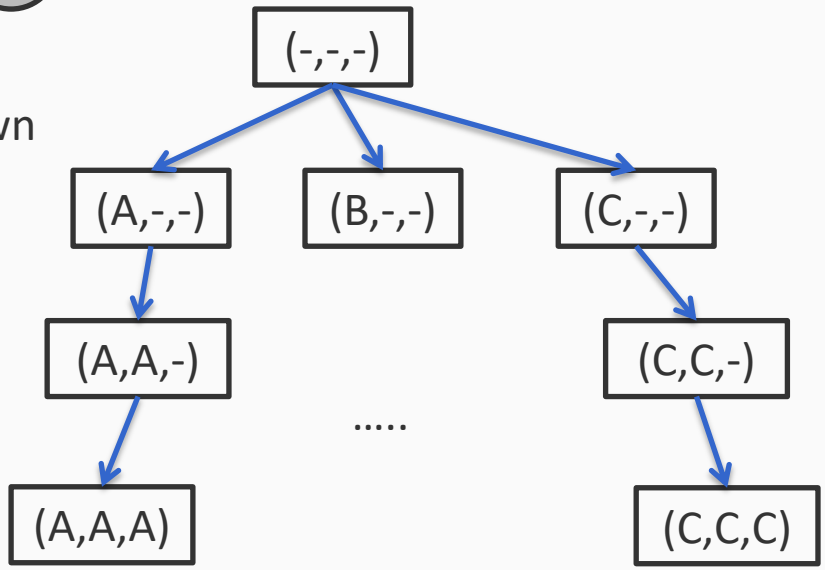

1st Framework:

LaSO: Learning as Search Optimization

[Hal Daumé III and Daniel Marcu, ICML 2005]

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The goal is to learn w. How?

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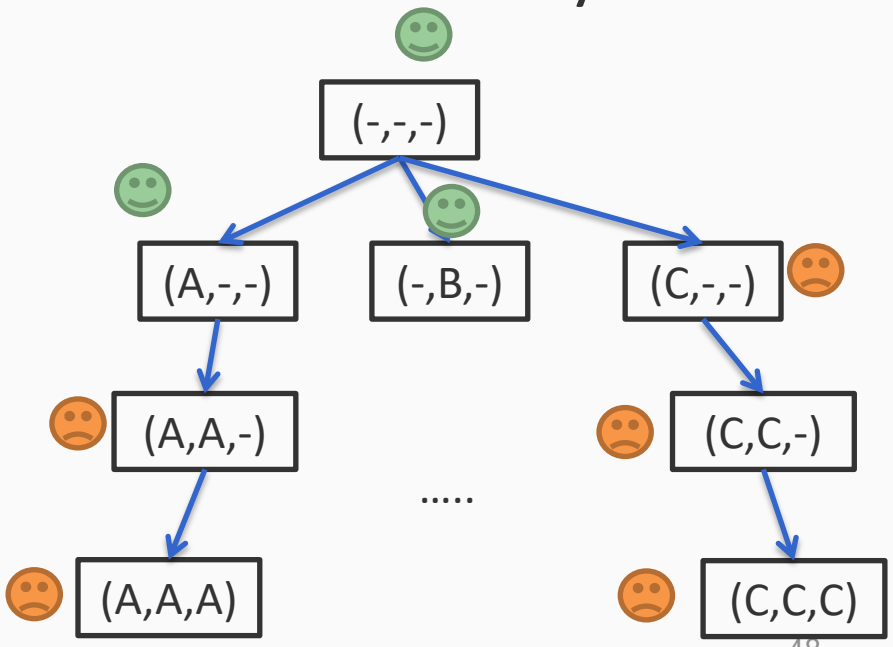

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  - Error type 1: none of the queue is y-good
  - Error type 2: the goal state is not y-good

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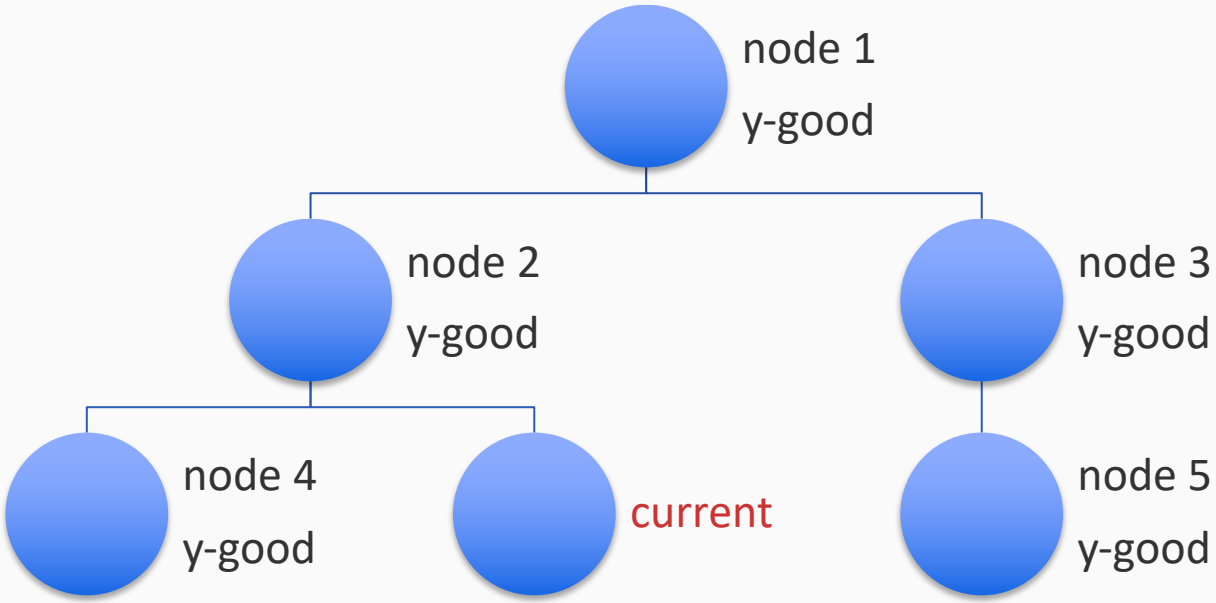

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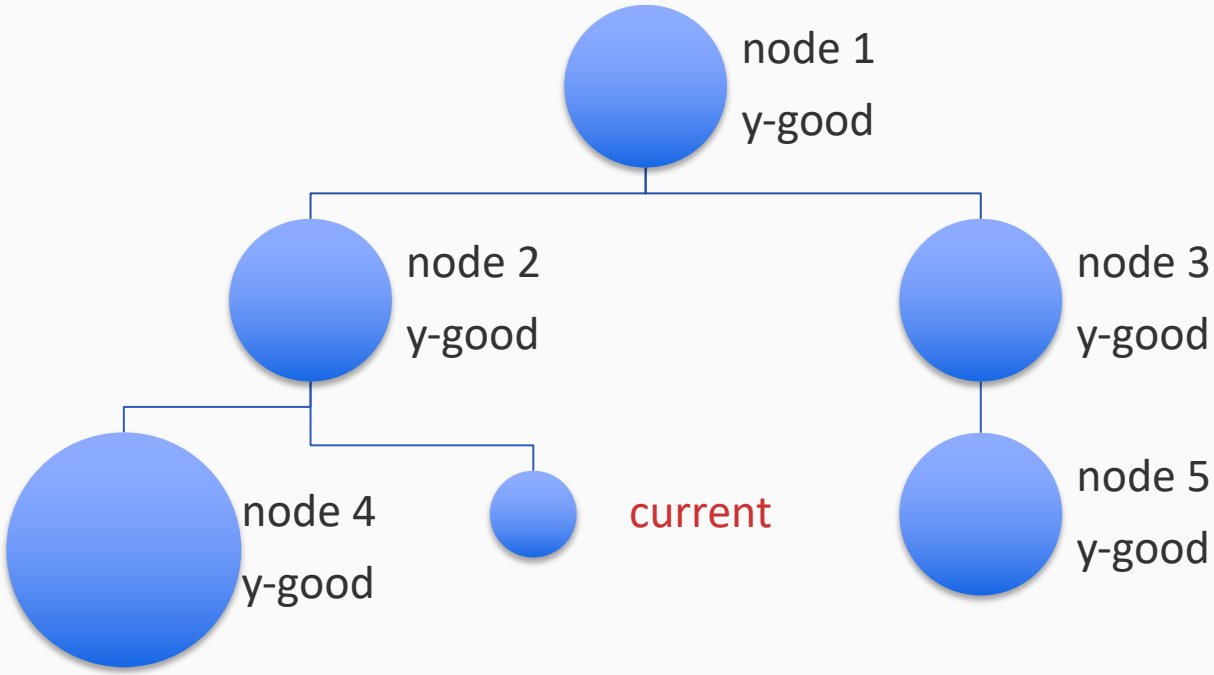

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Node 4 is the y-good *sibling* of the current node

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    else
        if GoalTest(*node*) **then return** *w*
        next = Result(*node*, Actions(*node*))
        nodes = *enqueue*(*problem, nodes, next, w*)

Parameter Updates

We need to specify *w* = update(*w, x, sibs, nodes*)

A simple perceptron-style update rule: w = w + Δ

$$\Delta = \sum_{n \in sibs} \frac{\Phi(x,n)}{|sibs|} - \sum_{n \in nodes} \frac{\Phi(x,n)}{|nodes|}$$

It comes with the usual perceptron-style mistake bound and generalization bound. (See references)

2nd Framework:

SEARN: Search and Learning

Hal Daumé III, John Langford, Daniel Marcu (2007)

Policy

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- A policy may be deterministic, or may contain some randomness. (More on this later)

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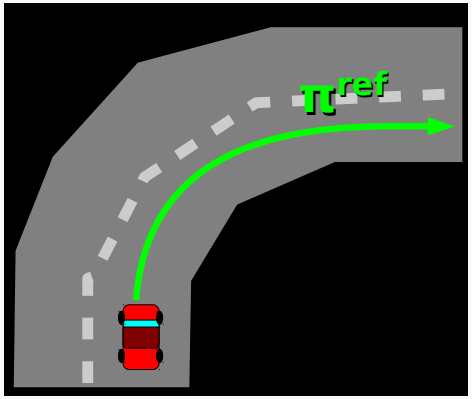

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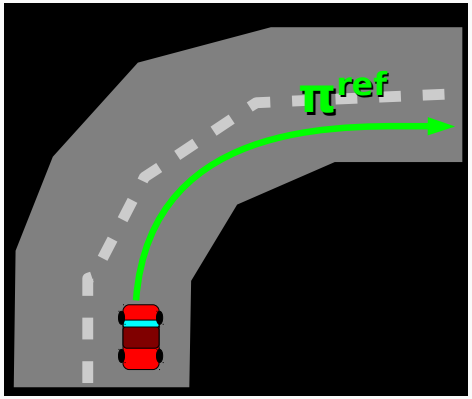

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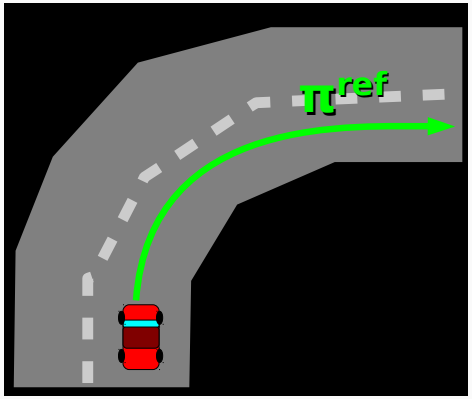

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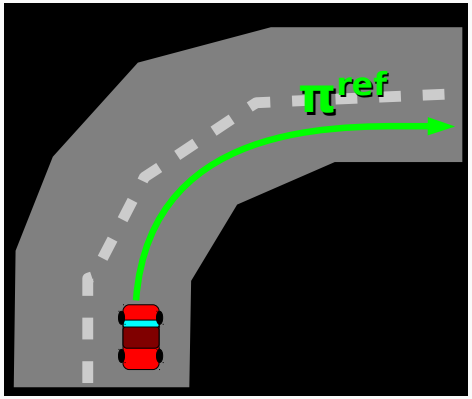

Suppose gold state is (A, B, C, A)
and we are at the state (A, C, -, -)
The reference policy tells us the next action is assigned C to the third slot.

Cost-Sensitive Classification

Suppose we want to learn a classifier h that maps examples to one of K labels

Standard multiclass classification

- Training data: Pairs of examples associated with labels
  - $(x, y) \in X \times [K]$
- Learning goal: To find a classifier that has low error
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Exercise: How would you design a cost-sensitive learner?

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SEARN uses a cost-sensitive learner to learn a policy

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1. Use the learned policy on initial state (-,…, -) to compute y_1
2. Use the learned policy on state (y_1, -,…,-) to compute y_2
3. Keep going until we get y = (y_1,…,y_n)

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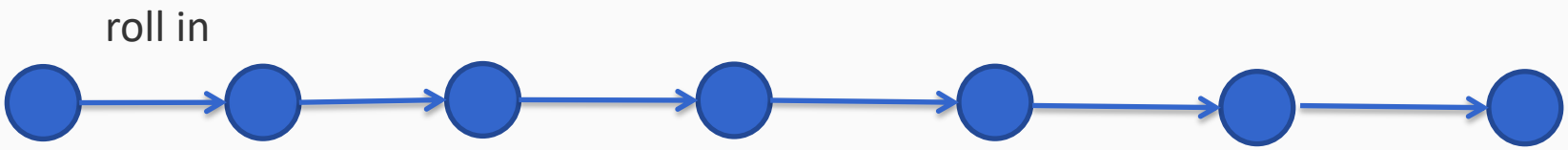
- The core idea in training is to notice that at each decision step, we are actually doing a cost-sensitive classification
- Construct cost-sensitive classification examples (s, c) with state s and cost vector c.
- Learn a cost-sensitive classifier. (This is nothing but a policy)

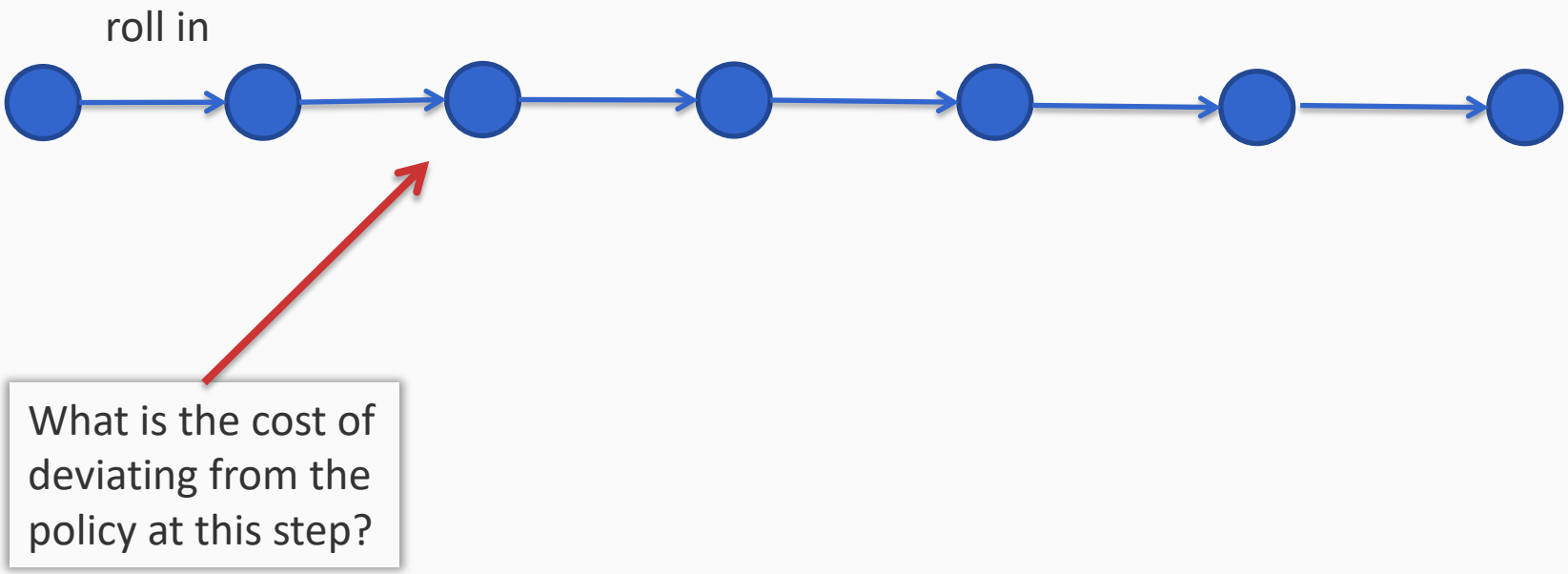
Roll-in, Roll-out

Roll-in, Roll-out



At each state, use some policy to move to a new state.

Roll-in, Roll-out



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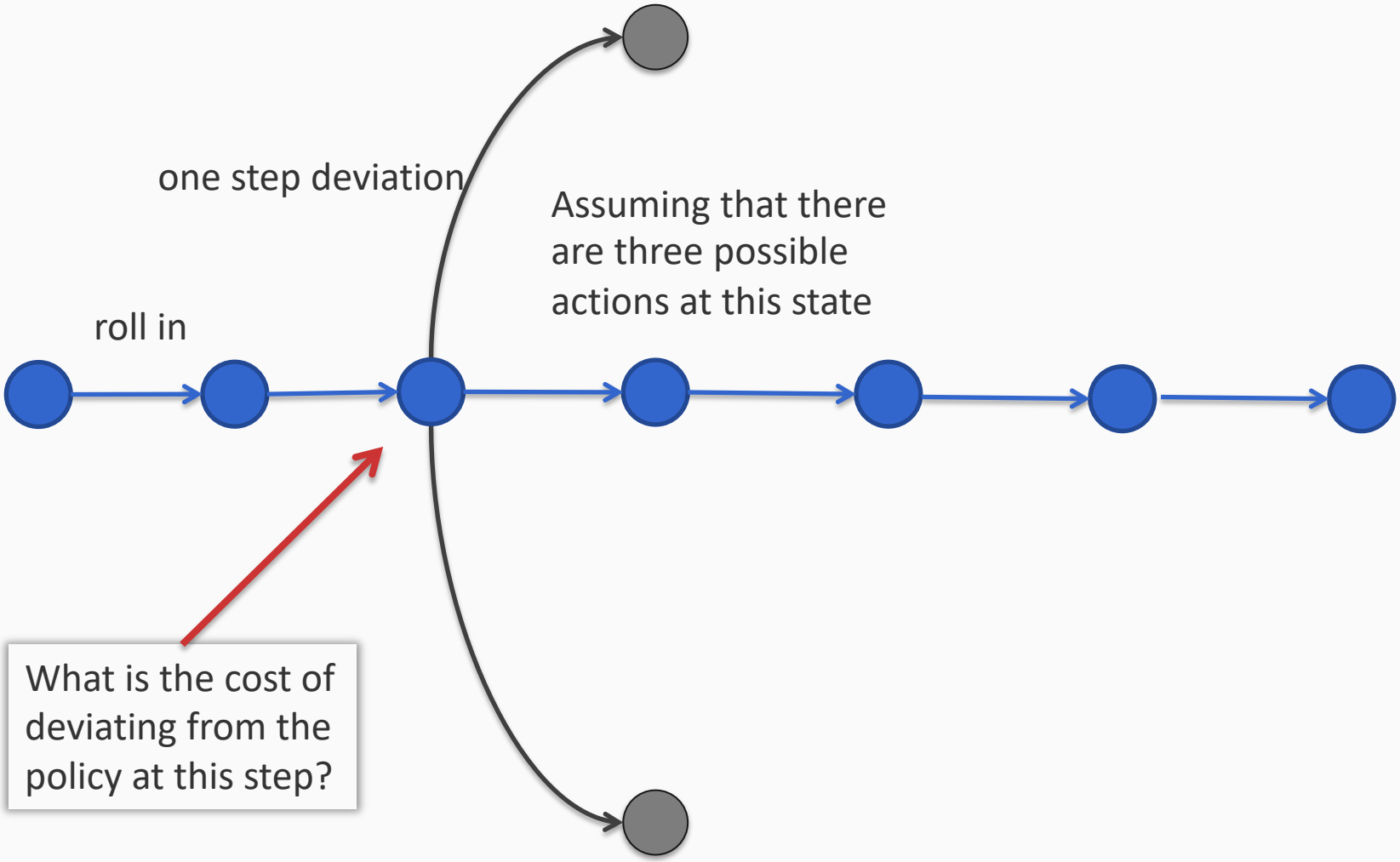

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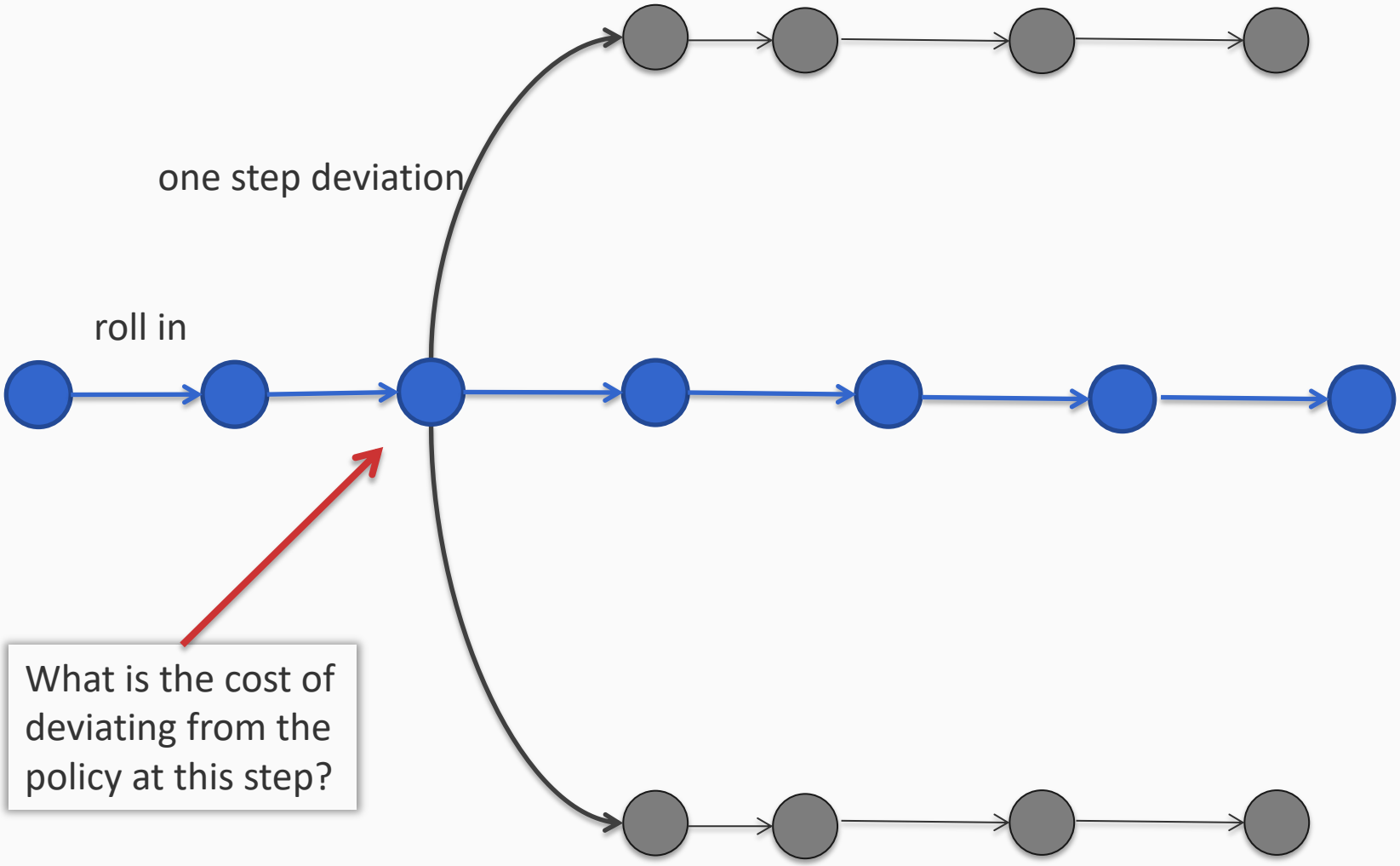

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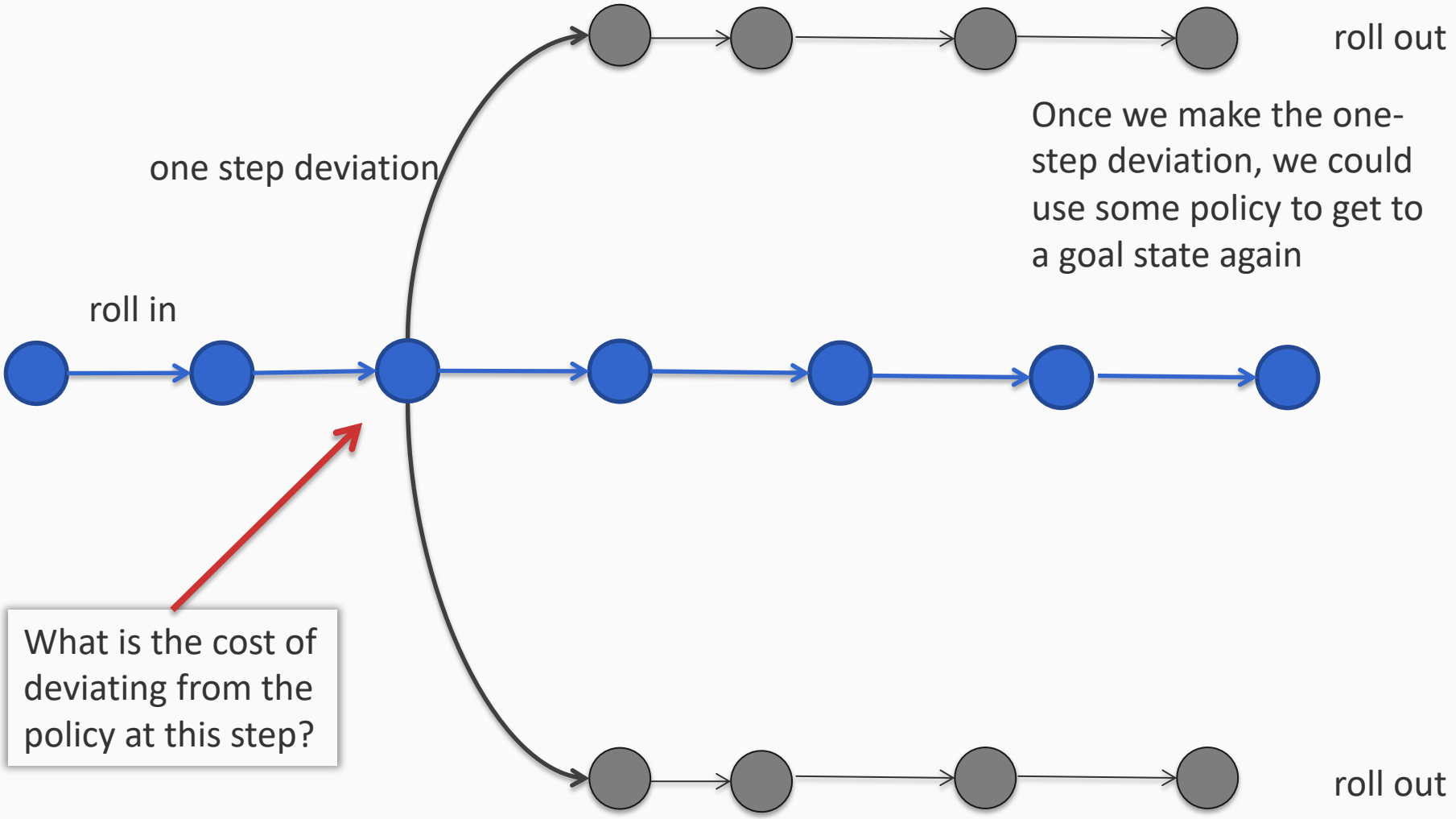

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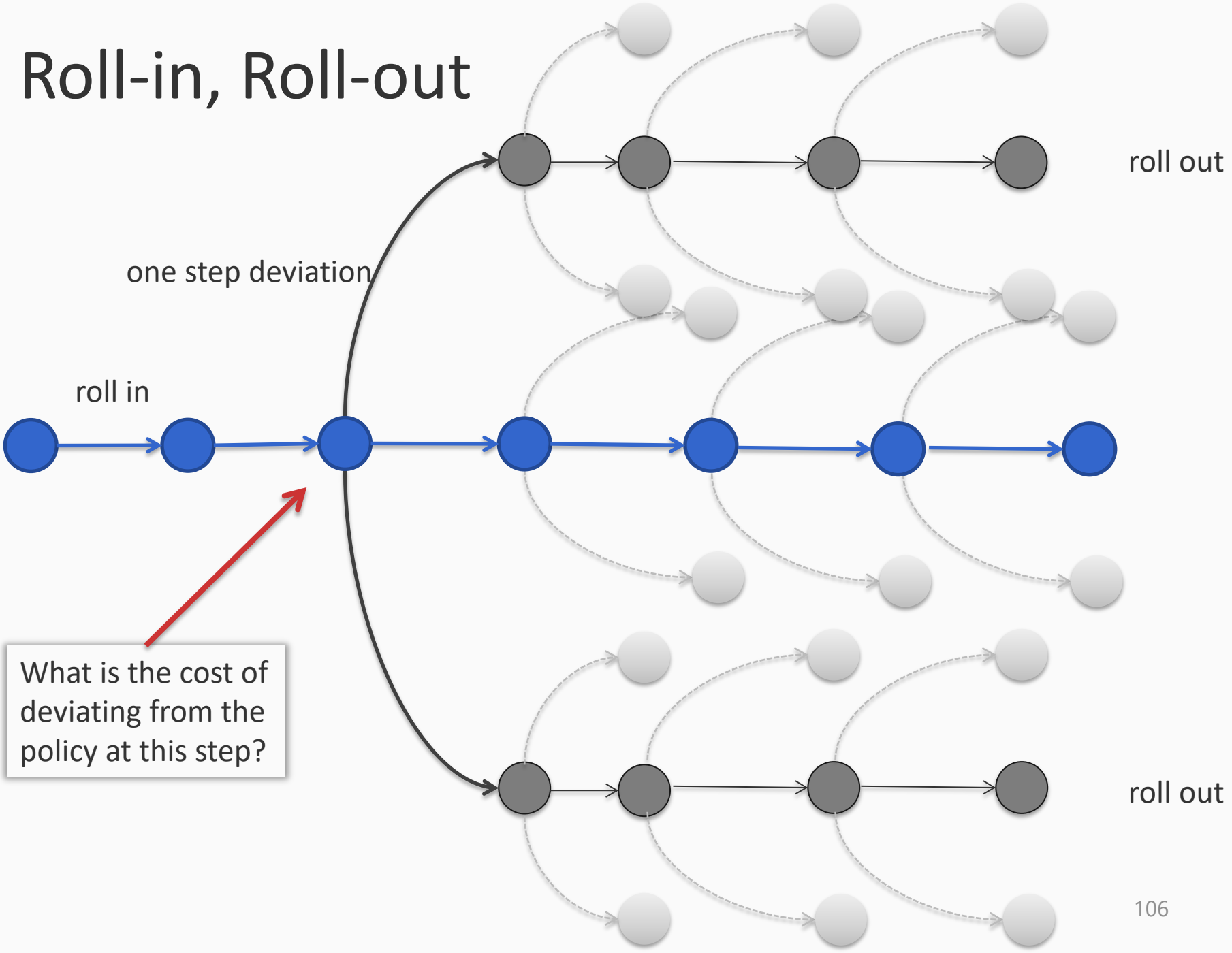
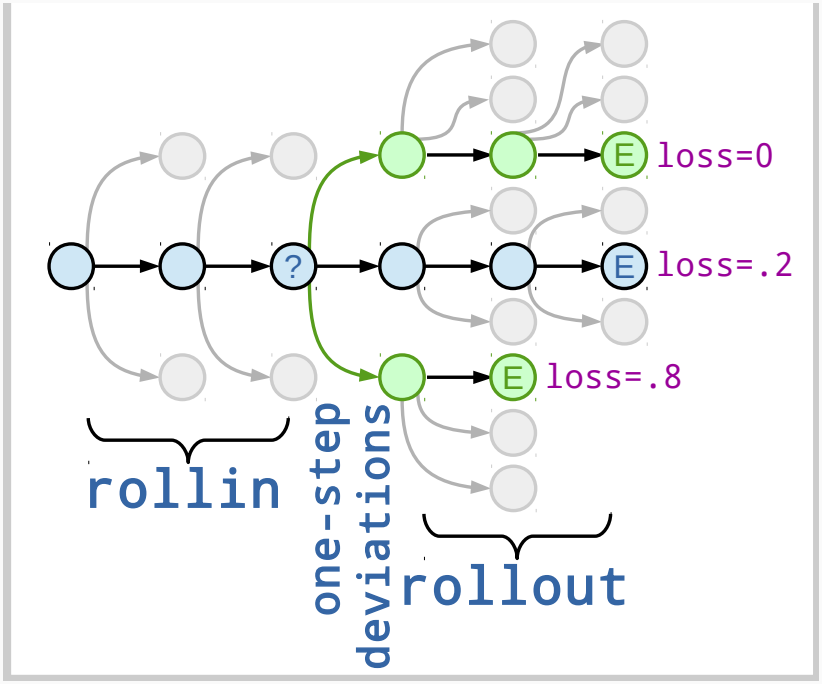
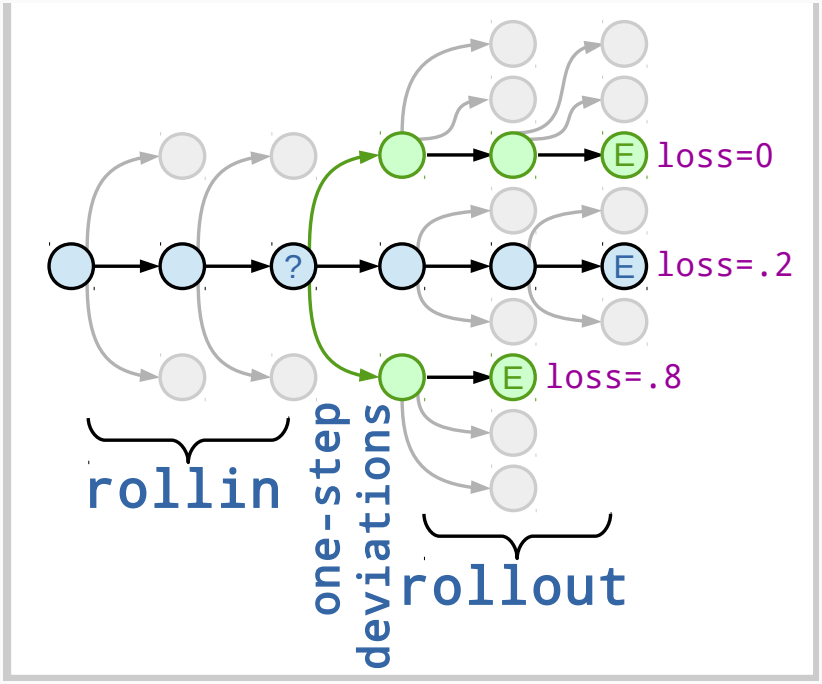


SEARN at training time (continued)



SEARN at training time (continued)



- Generate a search path

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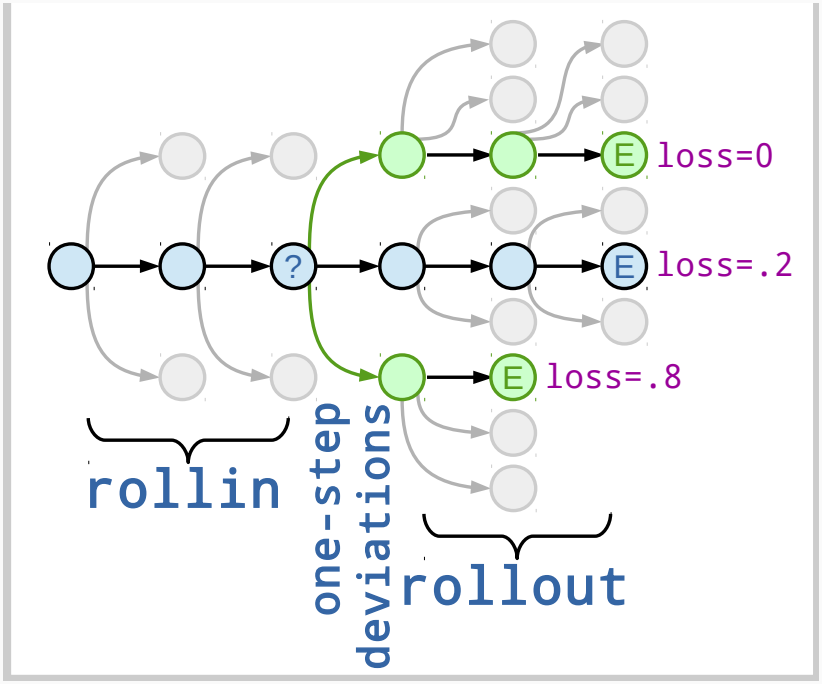

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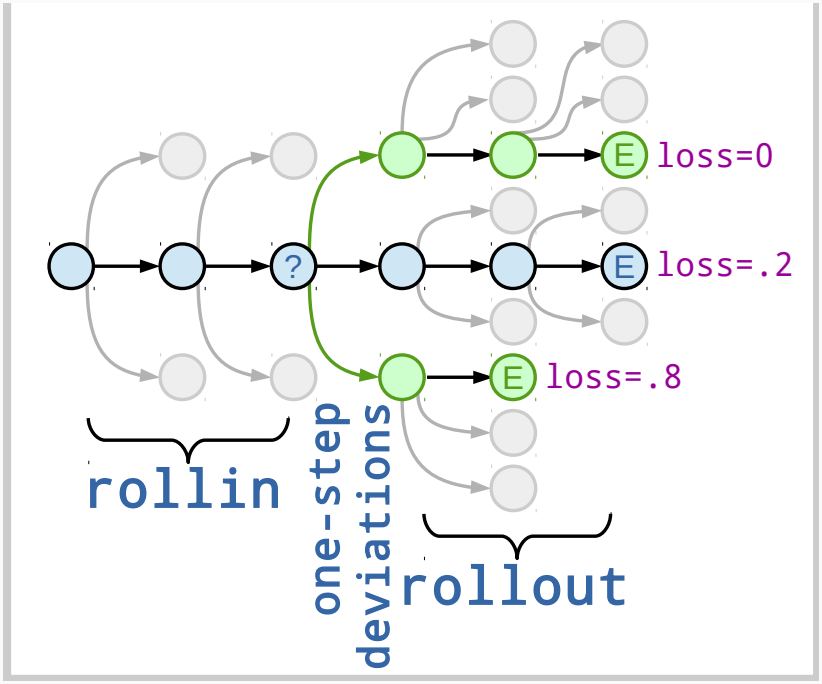
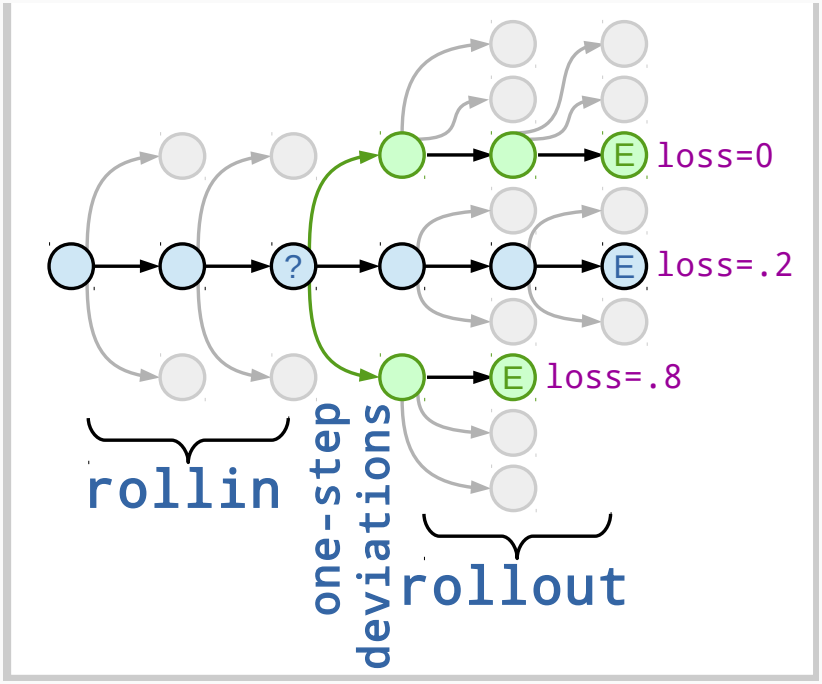


- Generate a search path
- Construct a cost-sensitive example: (?-state, c=(0, 0.2, 0.8))
- Do this for every step along the path

SEARN at training time (continued)



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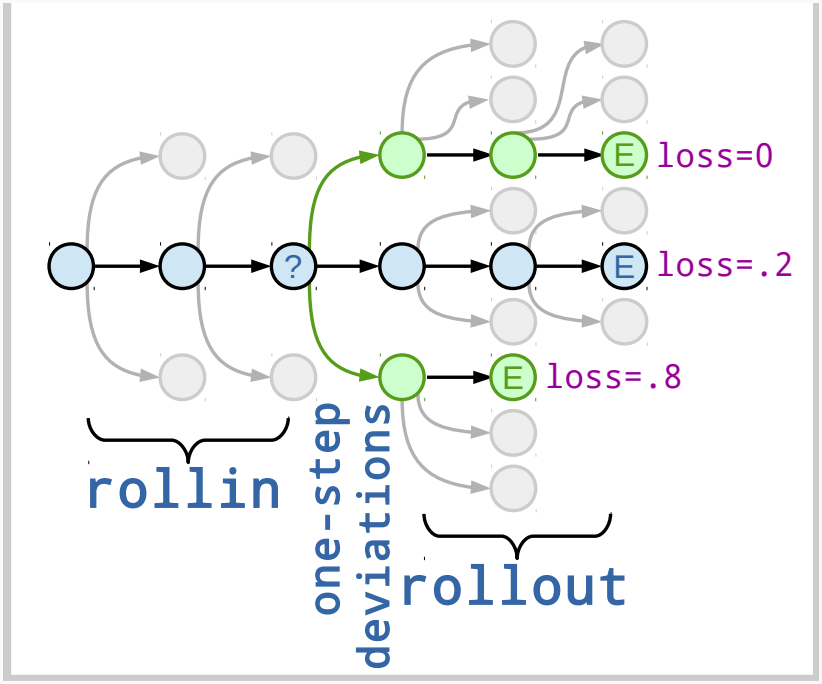

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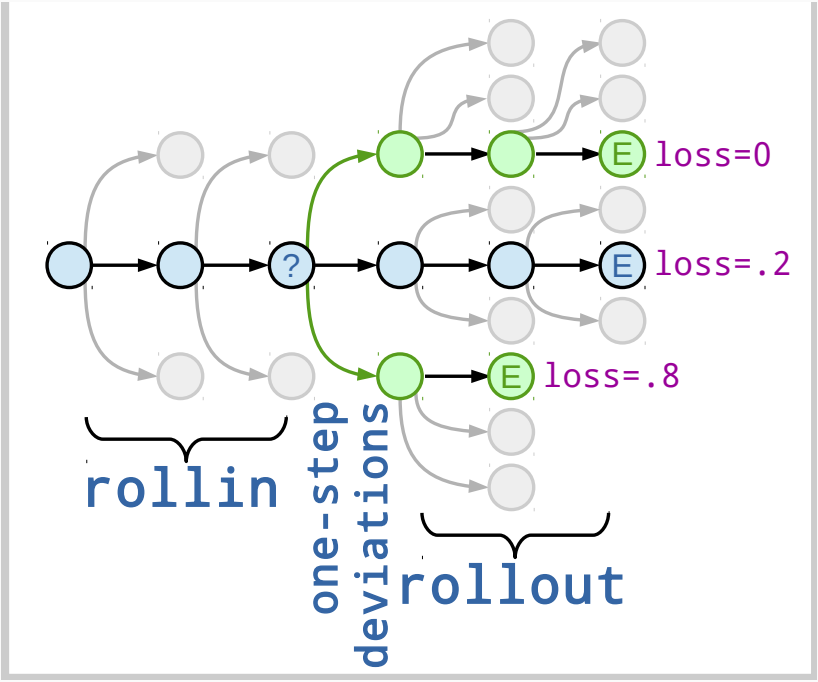

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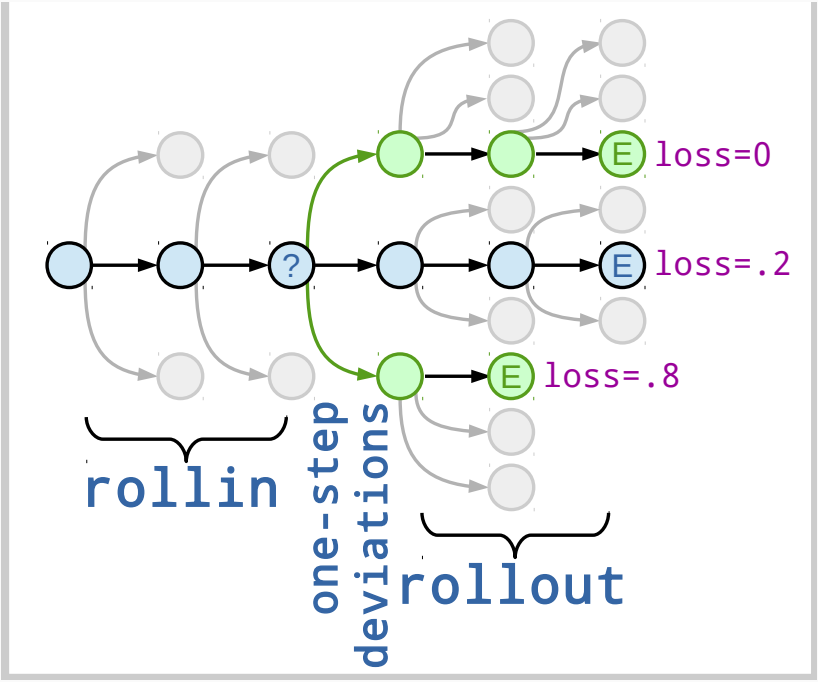

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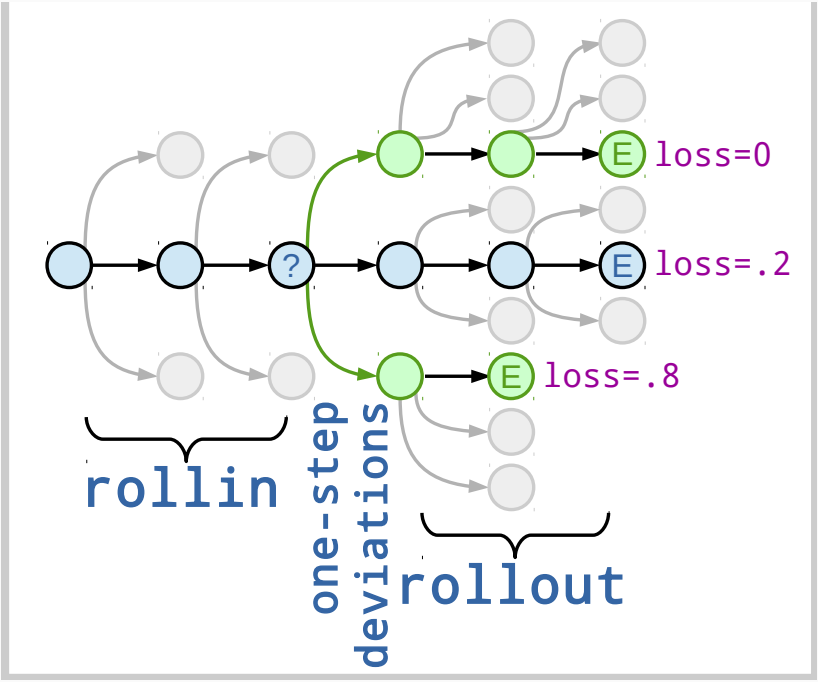

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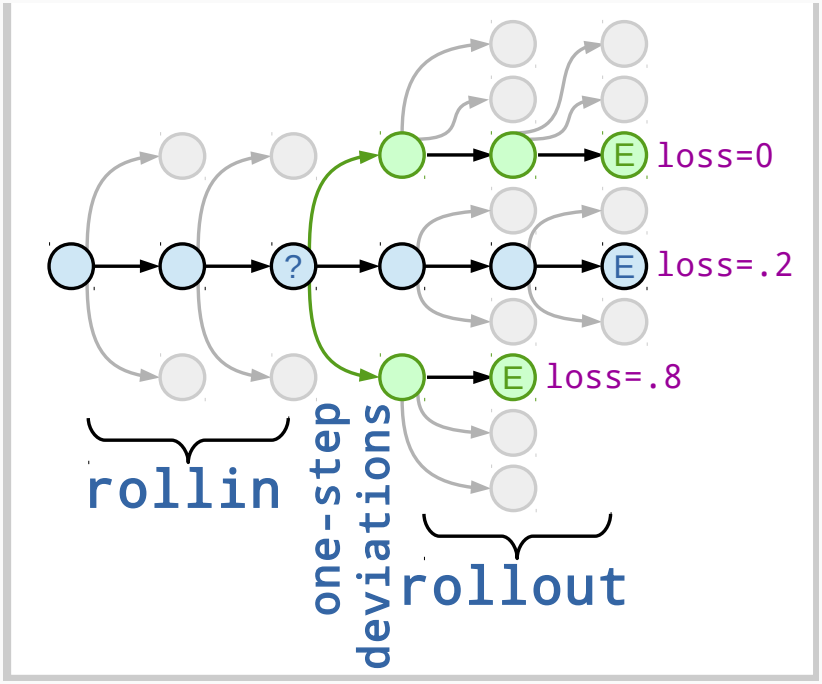

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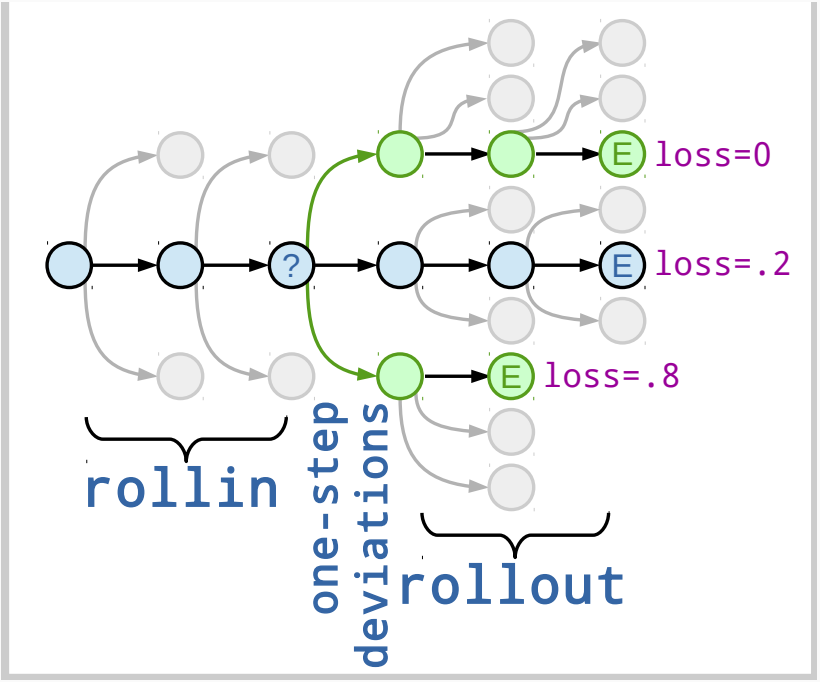

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- If h is deterministic:

$$l_h(c, s, a) = c_{y(s,a,h)} - \min_{a'} c_{y(s,a',h)}$$

SEARN at training time (continued)

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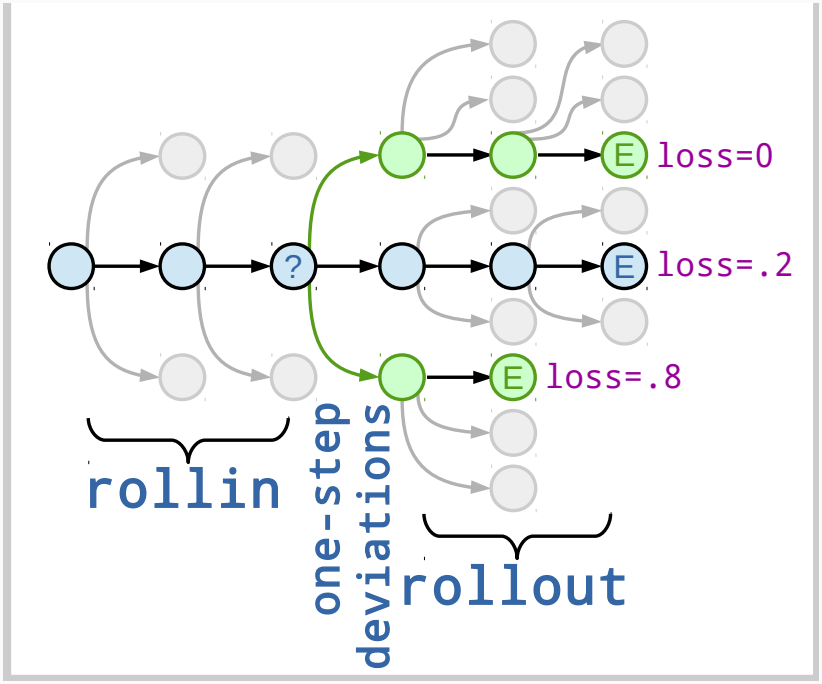

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$$l_h(c, s, a) = E_{y\sim(s,a,h)} c_y - \min_{a'} E_{y\sim(s,a',h)} c_y$$

SEARN at training time (continued)

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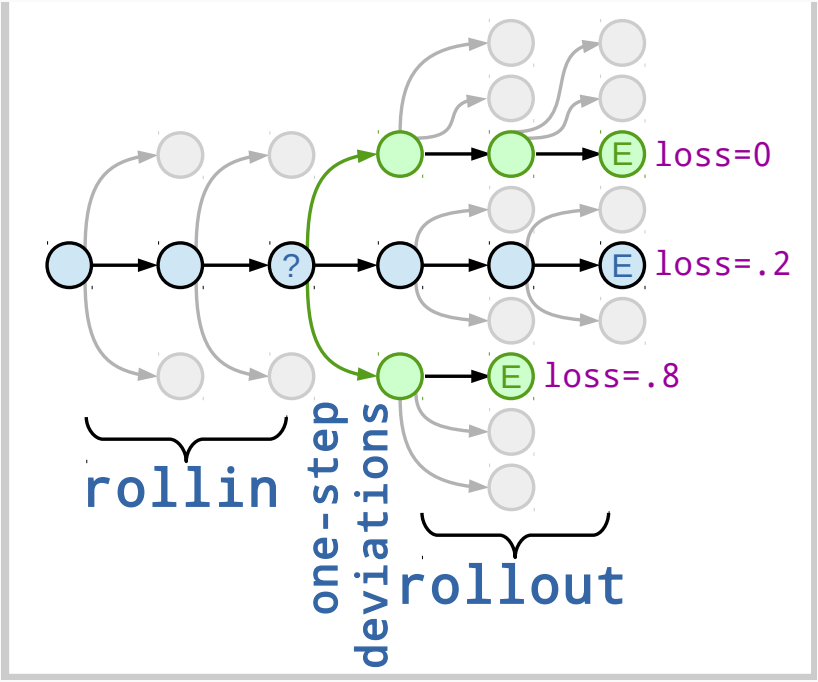

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The loss defined this way is called regret

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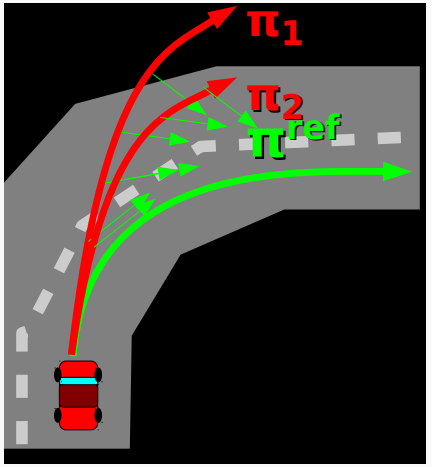
3rd Framework:

DAgger: Dataset Aggregation

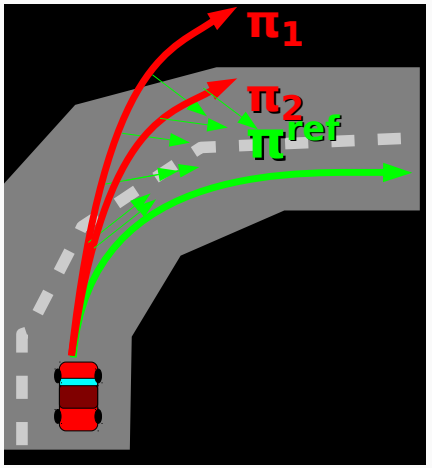
[Stéphane Ross, Geoffrey J. Gordon, J. Andrew Bagnell, 2011]

Dagger Algorithm (Simplified Version)



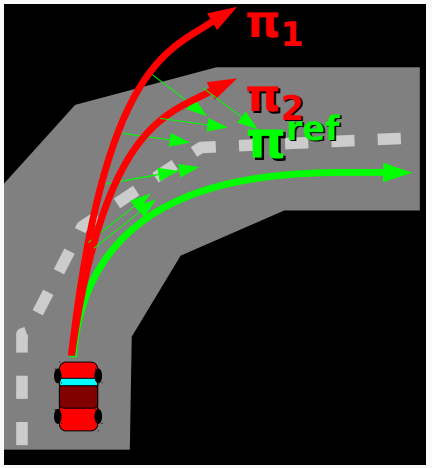
Dagger Algorithm (Simplified Version)

- Initialize Dataset $D = \emptyset$



Dagger Algorithm (Simplified Version)

- Initialize Dataset $D = \emptyset$
- Collect trajectories with reference policy π^{ref} (the expert)



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- Initialize Dataset $D = \emptyset$
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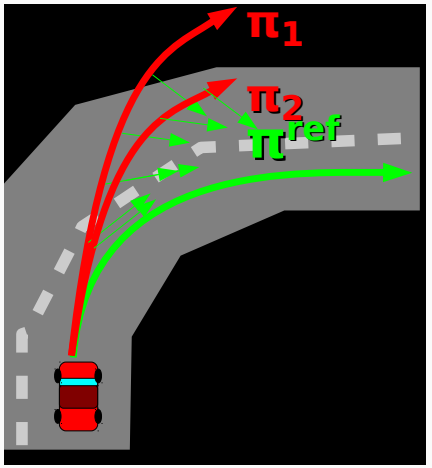

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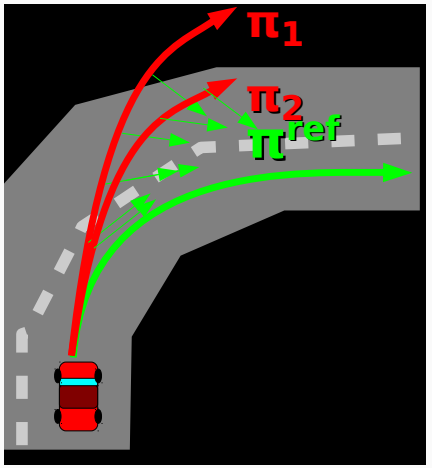

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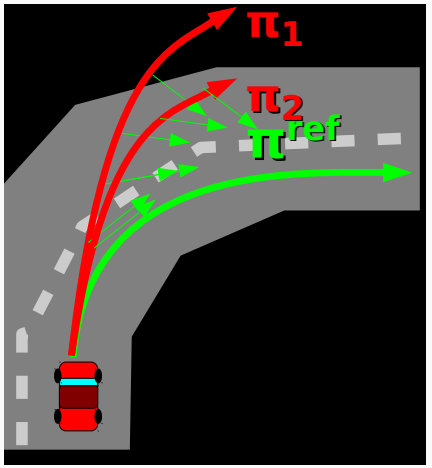
- Train π_1 on D



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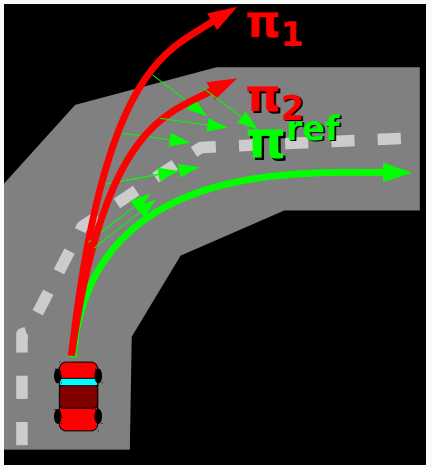
- Train π_1 on D
- Collect new trajectories with π_1



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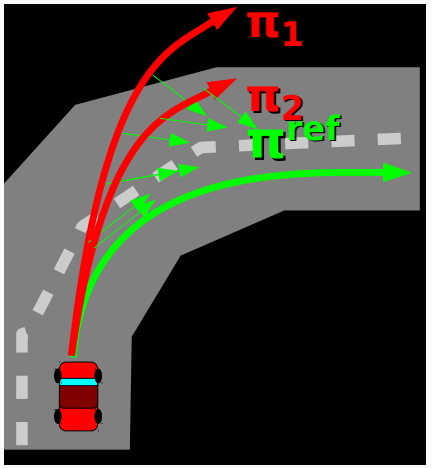
- Train π_1 on D
- Collect new trajectories with π_1
- New Dataset $D_2 = \left\{\left(s, \pi^{ref}(s)\right)\right\}$



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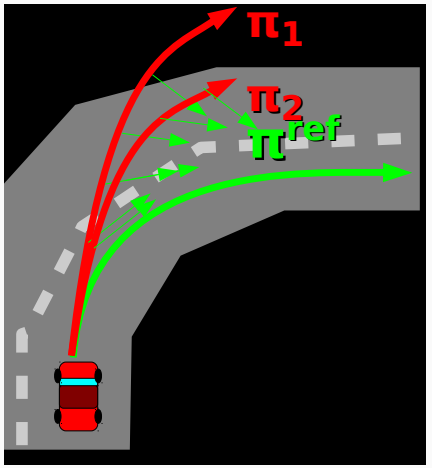
- Train π_1 on D

- Collect new trajectories with π_1
- New Dataset $D_2 = \left\{\left(s, \pi^{ref}(s)\right)\right\}$
- Aggregate Datasets $D = D \cup D_2$



Dagger Algorithm (Simplified Version)

- Initialize Dataset $D = \emptyset$
- Collect trajectories with reference policy π^{ref} (the expert)
- Dataset $D_1 = \left\{\left(s, \pi^{ref}(s)\right)\right\}$
- Aggregate Datasets $D = D \cup D_1$
- Train π_1 on D

- Collect new trajectories with π_1
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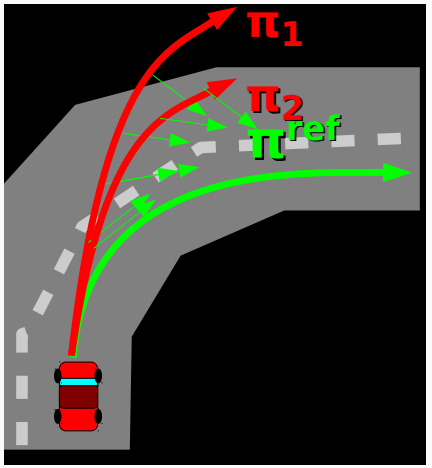
- Aggregate Datasets $D = D \cup D_2$
- Train π_2 on D



DAgger V.S. SEARN

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- Aggregate dataset

Other related algorithms

- Incremental Perceptron (2002)
  - Based on structured Perceptron
  - Instead of finishing inference during training, when inference makes its first mistake, stop and update parameters
- AggreVaTe: Aggregate Values to Imitate (2014)
  - Combines ideas from DAgger and SEARN
  - Cost-sensitive learning + dataset aggregation
- LOLS: Locally Optimal Learning to Search (2015)
  - What if the reference policy is not good?
  - Changes roll-outs to account for this

Learning to search: Summary

- Inference in structured prediction can be framed as search
  - Can we learn a model that explicitly helps inference navigate the search space?

- Several algorithms:
  - LaSO, SEARN, DAgger, etc
  - Often easy to implement with simpler building blocks
    - Can be the basis of a general purpose structured prediction framework