


# Neuro-Symbolic Modeling: Introduction



Does successful prediction imply understanding?



Date: May 28, 585 BCE  
Time: Late afternoon  
Location: Ancient Greece

# Thales of Miletus

mathematician, astronomer, philosopher

One of the earliest known successful  
eclipse predictions

Possibly learned how to do so from the  
Babylonian astronomers



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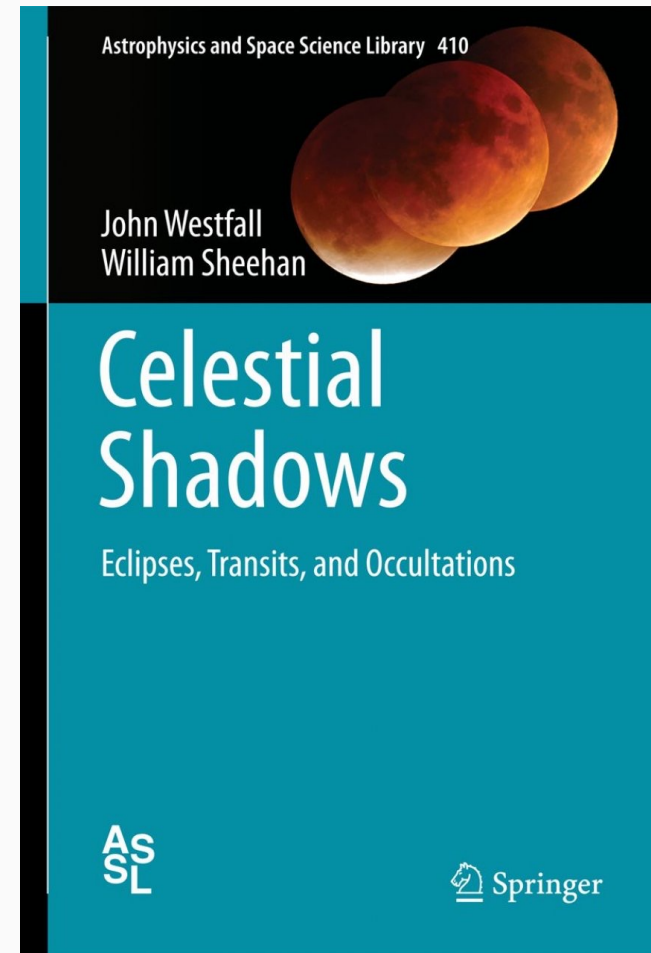
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Did Thales (and Babylonians)  
understand how eclipses work?



“The recognition that solar eclipses are caused by the Moon coming between the Earth and the Sun did not actually come until over a century [after Thales...].

Thus Thales **cannot** have predicted an eclipse in any modern sense.”



# Prediction sans understanding

Thales & co “data mined” celestial observations to find cycles in lunar and solar eclipses... but

...they had no clue about the positions of the earth, the moon and the sun!

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Spurious patterns → A recipe for pathological failures

We have increasingly sophisticated pattern recognition machines today

# Neural networks the default modeling tool today

They have demonstrated remarkable successes

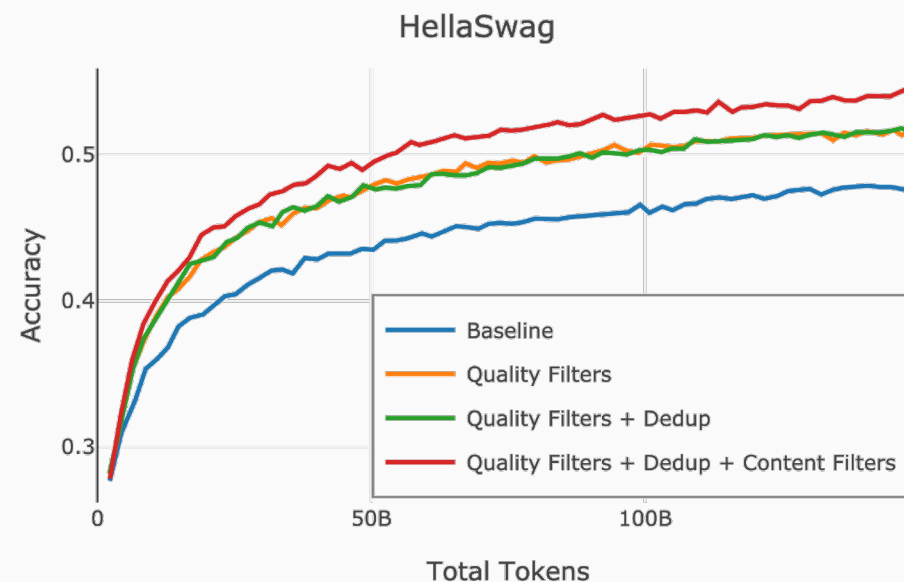
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- They need a lot of data!

| Source               | Doc Type       | UTF-8 bytes (GB) | Documents (millions) | Unicode words (billions) | Llama tokens (billions) |
|----------------------|----------------|------------------|----------------------|--------------------------|-------------------------|
| Common Crawl         | 🌐 web pages    | 9,812            | 3,734                | 1,928                    | 2,479                   |
| GitHub               | 📄 code         | 1,043            | 210                  | 260                      | 411                     |
| Reddit               | 💬 social media | 339              | 377                  | 72                       | 89                      |
| Semantic Scholar     | 🎓 papers       | 268              | 38.8                 | 50                       | 70                      |
| Project Gutenberg    | 📖 books        | 20.4             | 0.056                | 4.0                      | 6.0                     |
| Wikipedia, Wikibooks | 📖 encyclopedic | 16.2             | 6.2                  | 3.7                      | 4.3                     |
| <b>Total</b>         |                | <b>11,519</b>    | <b>4,367</b>         | <b>2,318</b>             | <b>3,059</b>            |



# Neural networks the default modeling tool today

They have demonstrated remarkable successes

But...

- They need a lot of data!
- Some results are not easy to explain.

Eight of the top ten models on the SUPERGLUE benchmark suite for text understanding outperform the human baseline!

*What does that mean?*

## Leaderboard Version: 2.0

|   | Rank | Name                       | Model                                 | Score |
|---|------|----------------------------|---------------------------------------|-------|
| + | 1    | Inspur Cloud               | Hairuo                                | 91.4  |
|   | 2    | JDExplore d-team           | Vega v2                               | 91.3  |
| + | 3    | Liam Fedus                 | ST-MoE-32B                            | 91.2  |
|   | 4    | Microsoft Alexander v-team | Turing NLR v5                         | 90.9  |
|   | 5    | ERNIE Team - Baidu         | ERNIE 3.0                             | 90.6  |
|   | 6    | Yi Tay                     | PaLM 540B                             | 90.4  |
| + | 7    | Zirui Wang                 | T5 + UDG, Single Model (Google Brain) | 90.4  |
| + | 8    | DeBERTa Team - Microsoft   | DeBERTa / TuringNLRv4                 | 90.3  |
|   | 9    | SuperGLUE Human Baselines  | SuperGLUE Human Baselines             | 89.8  |
| + | 10   | T5 Team - Google           | T5                                    | 89.3  |



Failures in reasoning and understanding

# Visual question answering

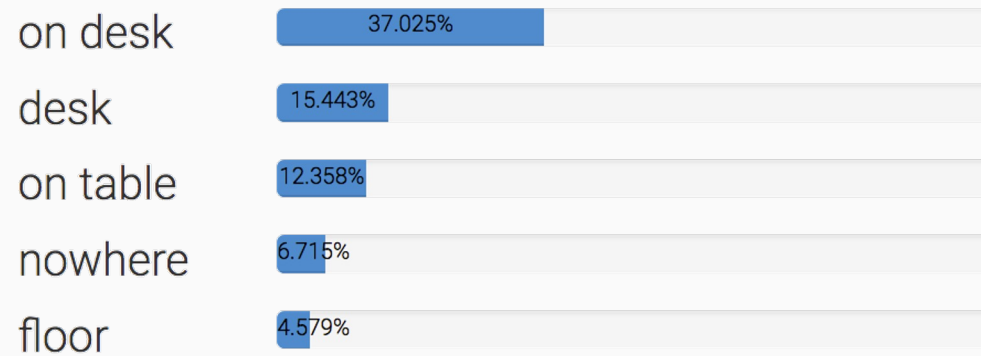


# Visual question answering



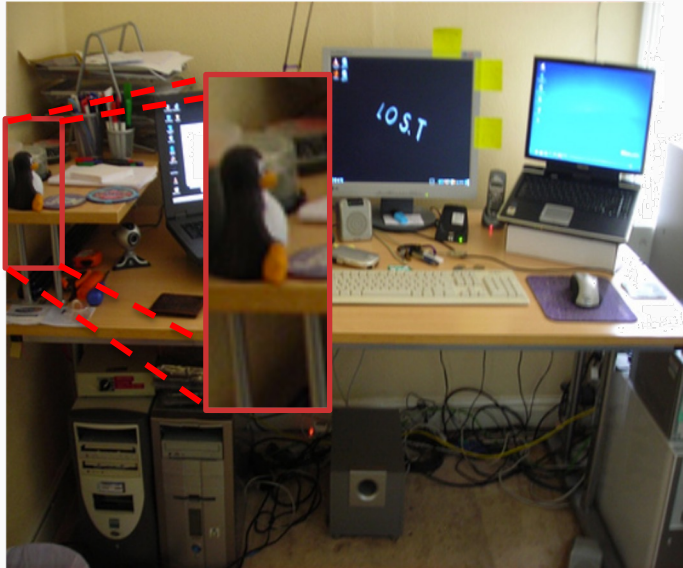
Where is the penguin?

Predicted top-5 answers with confidence:



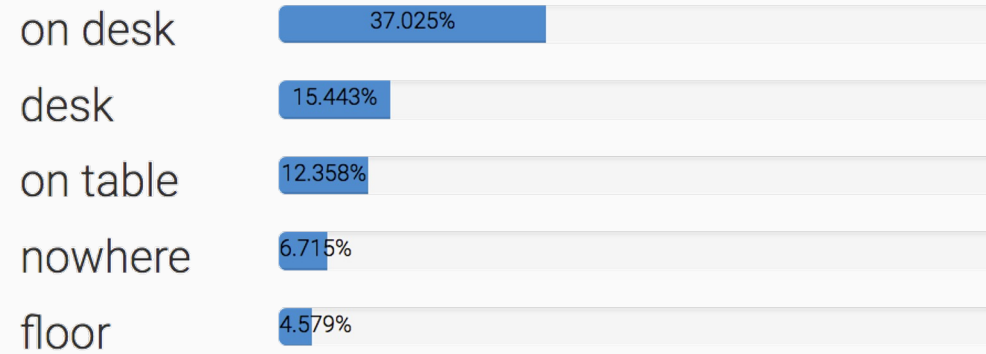


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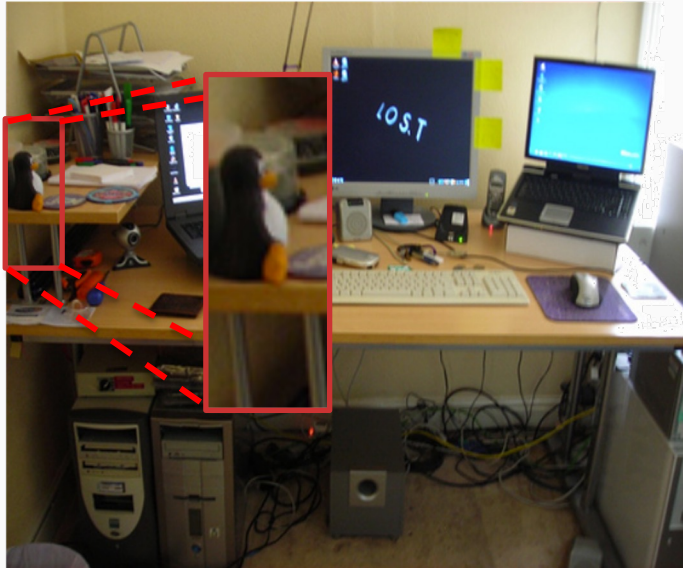


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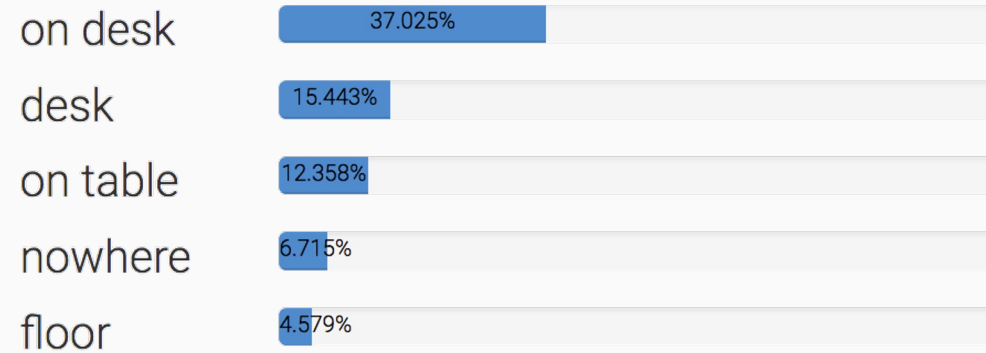
# Visual question answering



But can they really  
'read' images?

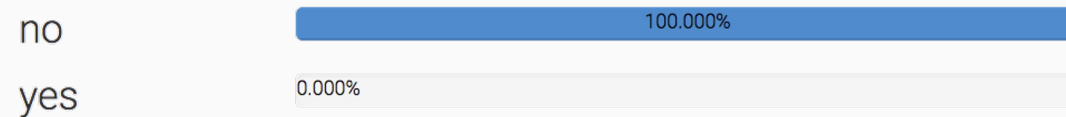
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Is there a penguin in the image?

Predicted top-5 answers with confidence:

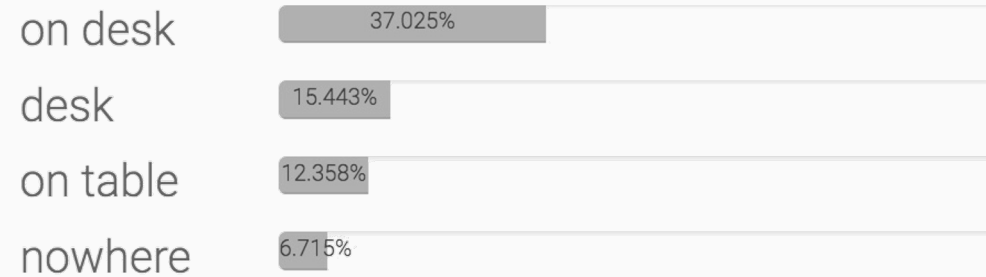


# Can neural networks 'read' images?



Where is the penguin?

Predicted top-5 answers with confidence:



Only one of these answers can be correct.

But the model is devoid of any such "theory"

Is there a penguin in the image?

Predicted top-5 answers with confidence:



## But how about more recent models?

How many letters are circled?



- ◆ The image shows the word "zoological" with the letter "o" circled. Therefore, there is **1** letter circled in the image.

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*Almost impressive but not quite right. The letter 'o' isn't circled*

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- ◆ The image shows the word "zoological" with the letter "o" circled. Therefore, there is **1** letter circled in the image.

Is the letter 'c' circled?

- ◆ **Yes**, the letter 'c' is circled in the image.

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Is the letter 'c' circled?

Given **this** claim...

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# Natural language inference

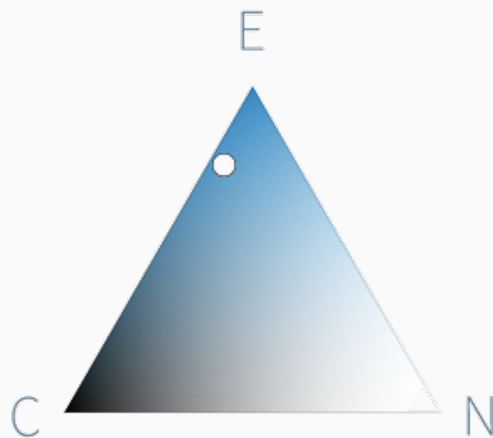
**Premise** Before it moved to Chicago, aerospace manufacturer Boeing was the largest company in Seattle.

**Hypothesis** Boeing is a Chicago-based aerospace manufacturer.

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| Judgment      | Probability |
|---------------|-------------|
| Entailment    | 75.6%       |
| Contradiction | 19.9%       |
| Neutral       | 4.5%        |

It is quite likely that the premise **entails** the hypothesis.

# *Can neural networks understand text?*

- P John is on a train to Berlin.
- H John is traveling to Berlin.
- Z John is having lunch in Berlin.

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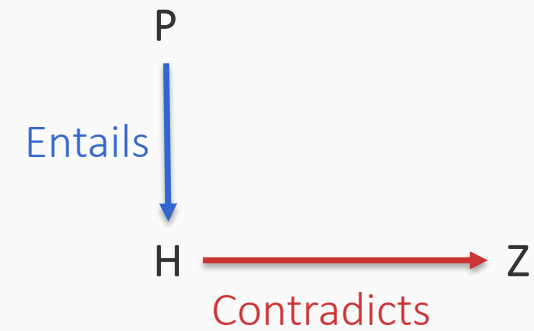
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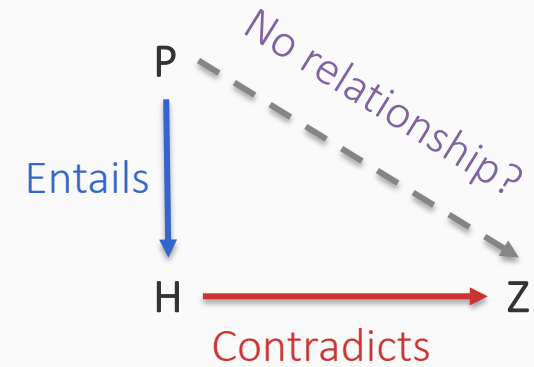
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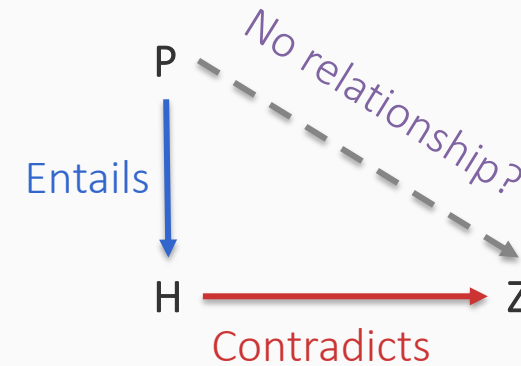
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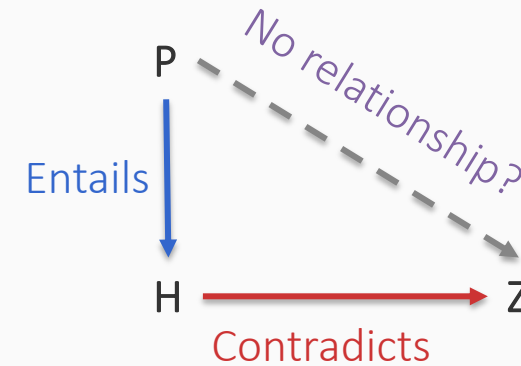


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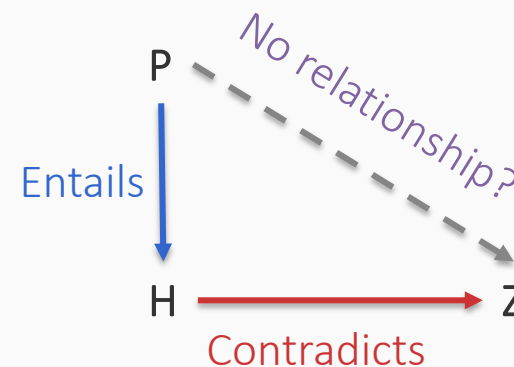
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Violates this invariant

If *P entails H* and *H contradicts Z*,  
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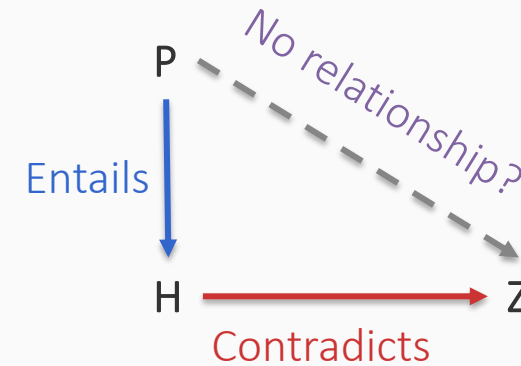
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A BERT-based model that gets ~90% on benchmark data violates this invariant on 46% of a large collection of sentence triples.

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Can today's neural networks use such "theory"  
in the form of invariant knowledge?

# Are we modeling problems in their full richness?

*Or: are we modeling for benchmark test sets?*

**Challenge:** Modeling strategies that depend on or expose a theory to reduce data dependence

(or at least assume the existence of a theory)

- Otherwise, we are just guessing equations involving high dimensional spaces, without understanding what they mean
- Thales, versus a modern understanding of eclipses  
Precise predictions via curve fitting versus a philosophy of the underlying phenomenon

# The problem with purely data-driven machine learning

Large models trained on large amounts of data can functionally approximate the intelligent behavior that led to the creation of the data

- Eg: Language models can produce token sequences that resemble language in style and content

*But explanation and understanding? We need systems that...*

- ...can be controlled via mechanisms other than just “add more data”
- ...can provide insight into their reasoning processes

A trip to the past

Good Old Fashioned Artificial Intelligence (GOFAI)

# Symbolic Artificial Intelligence

An agenda for artificial intelligence that focuses on knowledge representation and reasoning

Uses symbols (i.e. *discrete* labels), rules, and logic to represent knowledge

- Typically hand crafted rules

Well understood algorithms can reason about it

Rich human-auditable representations of the semantics and behavior of the programs

# Historically a dominant way to think about AI

1950s: The Dartmouth Summer Research Project, John McCarthy's work (e.g. Advice Taker)

1960s-70s: The development of rule-based systems

Two examples:

- ELIZA (Weizenbaum 1966): A chatbot that simulated a psychotherapist
- PARRY (Colby 1972): A chatbot that simulated a paranoid schizophrenic

1970s-1980s: The rise of logic programming (e.g., Prolog)





# A representative of this approach: Reasoning programs

Reasoning programs have various kinds of inputs and outputs (arrays to represent images, sequences for utterances, parse trees for sentences, etc)

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- Data structures, agent goals and sub-goals, rules governing behavior

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- Data structures, agent goals and sub-goals, rules governing behavior

The program is a *deduction engine* that

- tries to find strategies of action that it can prove will solve a problem, and
- on finding one, executes it

# An example from McCarthy (1959)

The situation: “Assume that I am seated at my desk at home and I wish to go to the airport. My car is at my home also.” What should I do?

Answer: “walk to the car and drive the car to the airport”

How is this represented symbolically?

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“Assume that I am seated at my desk at home and I wish to go to the airport. My car is at my home also.” What should I do?  
Answer: “walk to the car and drive the car to the airport”

First, the facts of the problem

*at(I, desk)      want(at(I, airport))*

*at(desk, home)*

*at(car, home)*

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*at(home, county)*

*at(airport, county)*

*Where did these come from?*

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The “at” predicate is transitive

$at(x, y), at(y, z) \rightarrow at(x, z)$

*This holds for any  $x, y, z$ .*

*But where did this rule come from?*



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Define feasibility of walking and driving

$walkable(x), at(y, x), at(z, x), at(I, y) \rightarrow can(go(y, z, walking))$

$drivable(x), at(y, x), at(z, x), at(car, y), at(I, car) \rightarrow can(go(y, z, driving))$

Something to consider: **Do we really think about this so explicitly?**

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Answer: “walk to the car and drive the car to the airport”

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$at(I, desk) \quad want(at(I, airport))$

$at(desk, home) \quad walkable(home)$

$at(car, home)$

$drivable(county)$

$at(home, county)$

$at(airport, county)$

This is not stated in the question, but we have to assume this. Which means, it needs to be explicitly represented. Both a strength and a weakness.

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And this goes on for a couple of pages...

The “at” predicate is transitive

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# The problem with GOFAI

Worked well, but *only* for a very small set of examples

Did not handle uncertainty and noise well

Took several years of human effort to create

And still did not generalize

Because too many hand-crafted rules

Hand-crafted rules do not really reflect how complex phenomena like language and vision work in practice!

# Neural networks versus Symbolic AI

## Neural networks

The best pattern recognition engines today

## Symbolic AI

Historically well developed and has deep algorithmic understanding

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✓ Handles uncertainty

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- ✓ Use raw data to improve behavior

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What do we want: Best of both worlds

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What do we want: Neuro-Symbolic Models

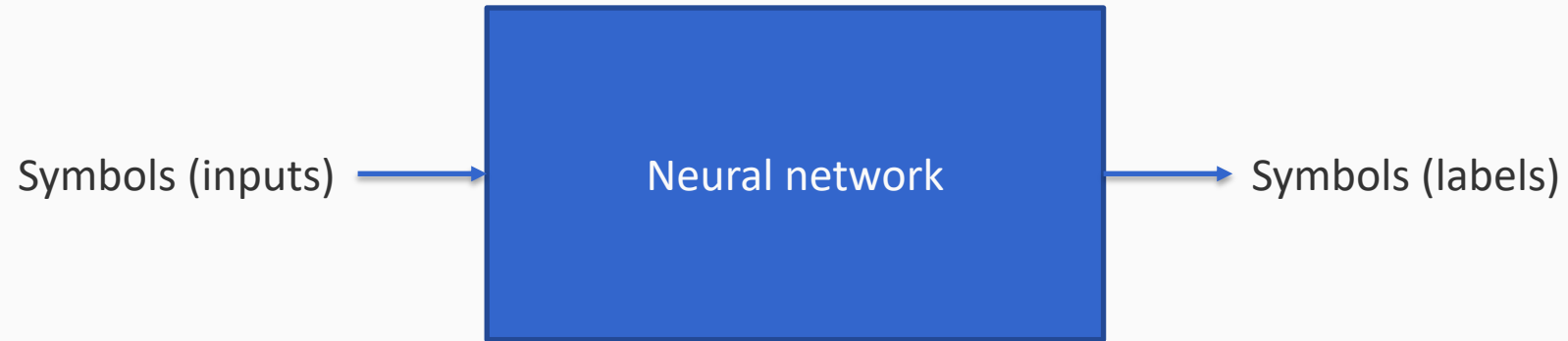
# Neural network + symbolic programs

Neural networks involve vectors and differentiable functions

Rules are symbolic objects (i.e. consisting of discrete decisions)

How do they interface with each other?

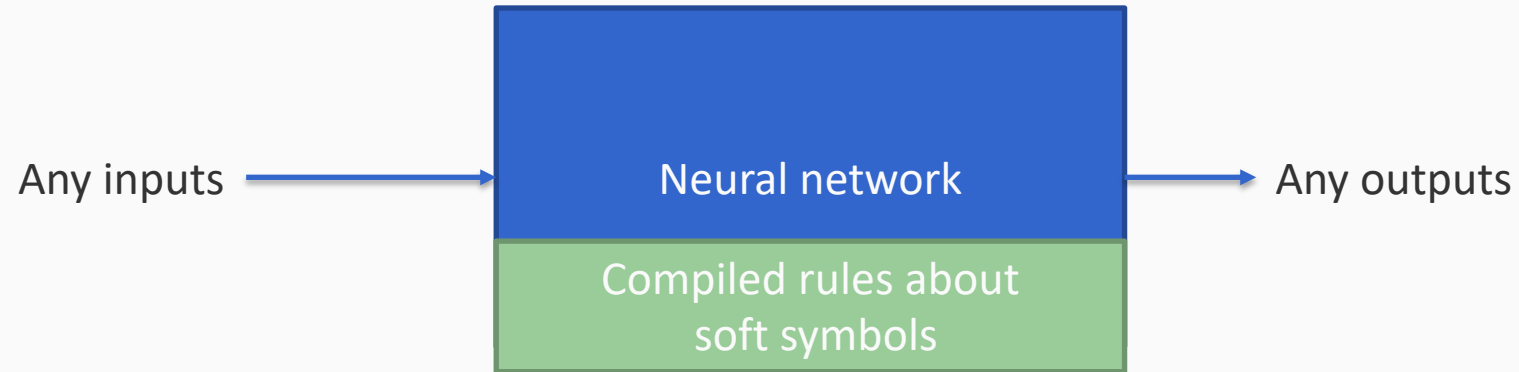
# Neuro-symbolic interfaces



Convert symbolic inputs into vectors, operate with them and produce symbolic outputs

Standard neural networks do this

# Neuro-symbolic interfaces

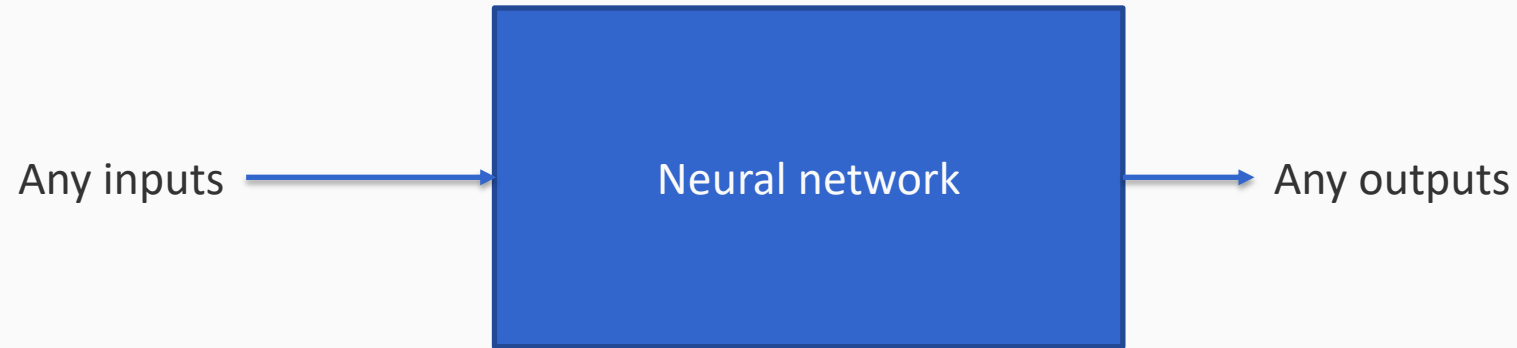


Treat some nodes within a neural network as soft symbols

Compile symbolic rules (i.e. constraints) into neural network submodules about these symbols to augment the network architecture



# Neuro-symbolic interfaces

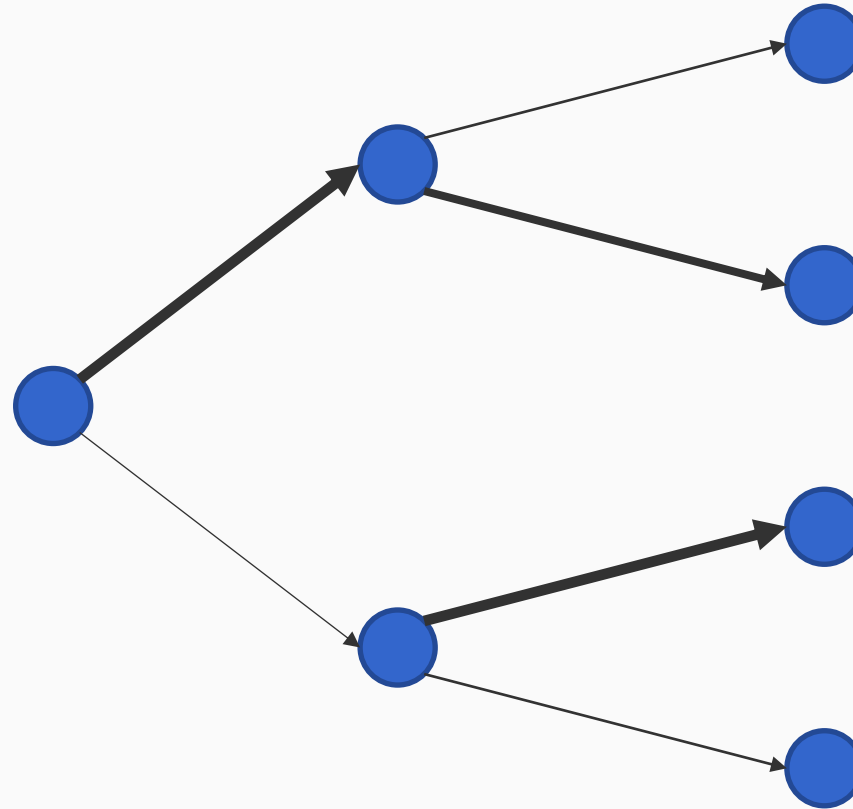


Training: data loss + compiled(rules)

Symbolic rules (about outputs or both inputs and outputs) compiled into a form that augments the training process

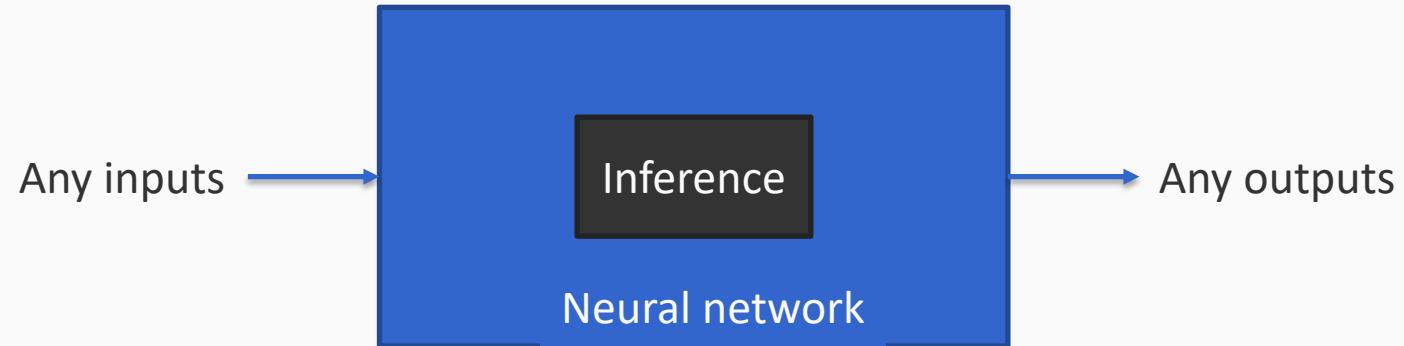
Implicit claim: Neural networks can internally represent the rules. *Is this valid?*

# Neuro-symbolic interfaces



Use a neural network to guide a program that navigates a symbolic space by scoring paths (e.g. game playing, combinatorial inference, etc)

# Neuro-symbolic interfaces



Symbolic reasoning inside a neural network

*Differentiability is an issue.*

# Neuro-symbolic interfaces



Convert symbolic inputs into vectors, operate with distributions over labels

Apply probabilistic inference (with symbolic knowledge) over the top of network outputs to produce final outputs: *Structured prediction*

What we will cover this semester

# Semester overview

## Part 0: Introduction

- Review of neural networks
  - The computation graph abstraction and gradient-based learning
  - Special neural networks (e.g. transformers)
- Review of symbolic logic
  - propositional logic and SAT
  - tractable representations
  - knowledge compilation

# Semester overview

## Part 1: The “logic-as-loss” approach

- The idea that logic can be compiled into loss functions
- Two high level strategies for “logic-as-loss”
  - Weighted model counting, semantic loss
  - Soft logic, t-norms
- Strengths and limitations of this strategy

# Semester overview

## Part 2: The “logic-as-network” approach

- The idea that logic can be compiled into neural networks
- Using soft logic for this
- Strengths and limitations of this strategy



# Semester overview

## Part 2: The “structured prediction” approach

- Structured prediction and training
- MAXSAT based methods
- Integer programming based methods
- Differentiable combinatorial optimization

# Semester overview

## Part 3: The “Reinforcement Learning” approach

- Reinforcement learning introduction
- The REINFORCE algorithm
- Applications with black-box symbolic programs

# Semester overview

## Part 4: Case studies with neuro-symbolic modeling

- We will encounter several examples during the previous sections
  - +
    - Incorporating human preference feedback in large language models  
And derive standard named algorithms as special cases)
- Your favorite topic
  - Let me know