Logic as Loss: Overview

THE UNIVERSITY OF UTAH

Logic as loss: The setting

- Suppose we would like to train one or more neural networks on some data
  - Each of them can take different kinds of inputs to produce their own discrete outputs

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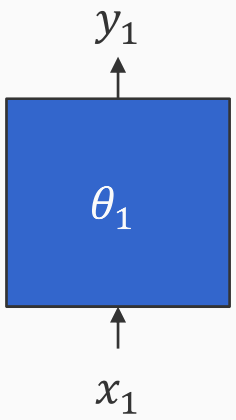
$$P(\, y_1 \mid x_1, \theta_1 \,)$$



This network takes inputs X_1 and produces a distribution over labels Y_1

Its architecture and parameters are defined by θ_1

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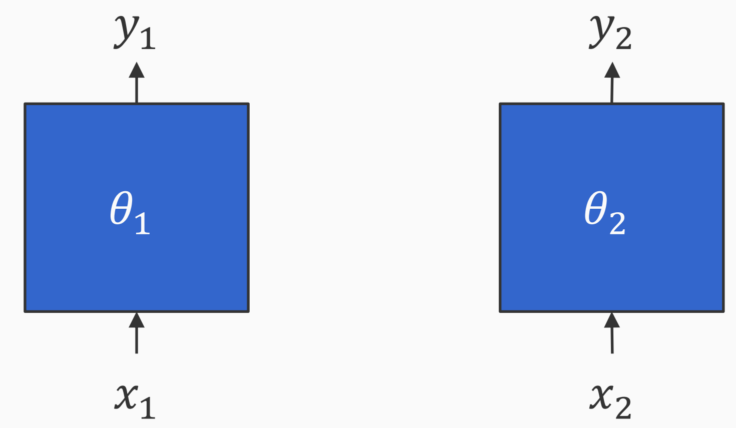

This network takes inputs X_2 and produces a distribution over labels Y_2

Its architecture and parameters are defined by θ_2

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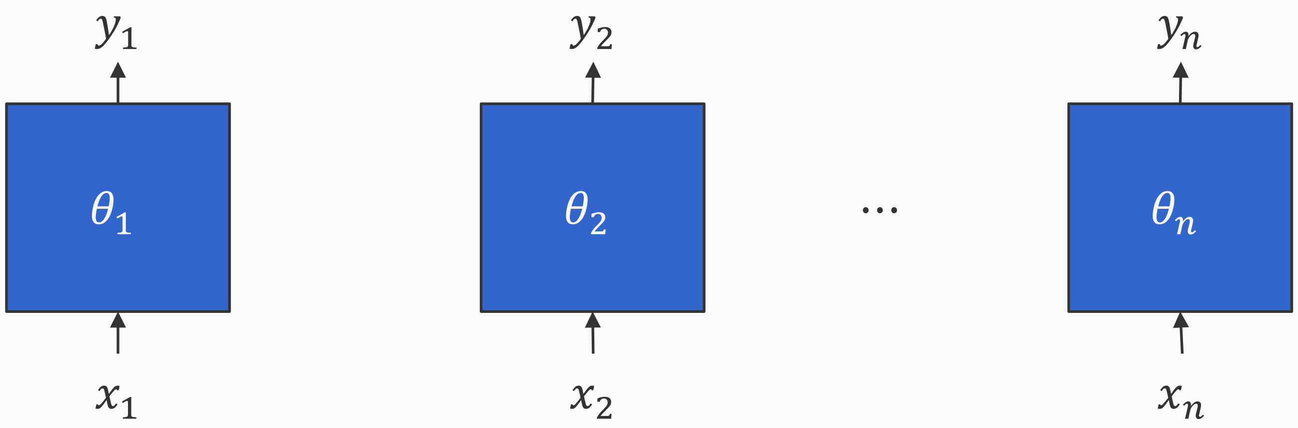
$$P(y_1 \mid x_1, \theta_1), P(y_2 \mid x_2, \theta_2), \cdots, P(y_n \mid x_n, \theta_n)$$



This network takes inputs X_n and produces a distribution over labels Y_n

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Important:

These n different networks and their inputs/outputs need not be different

- Maybe they are the same model called on different inputs
- Maybe they are different models given to the same network
- Or other combinations

What we have are n different conditional distributions over outputs given inputs

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How do we train these networks on their respective datasets?

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One answer: Minimize cross-entropy loss over the data

The loss penalizes models that assign low probabilities to observed data

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Recall that we can write classification tasks as predicates

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$\text{Label}(X_1, Y_1)$

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Corresponding to the networks, we have predicates that serve as a vocabulary for rules

- Suppose we want our models to satisfy some *invariant property* that is written in terms of these predicates *in addition to modeling the data*

How do we proceed?

The idea of the “logic as loss” framework

What we want of our models

Fit the available training data

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Minimize Data loss (e.g. cross entropy)

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What we want of our models

Fit the available training data → Data loss (e.g. cross entropy)

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Satisfy invariant properties about predictions → Constraint loss

Minimize Data loss (e.g. cross entropy) Constraint loss

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penalty for
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The intuition

We want to find models that maximally satisfy the constraints

If the constraint losses are well defined to capture the logic, models that violate constraints will have higher loss

So it is better to minimize the constraint loss

Labeled examples are also predicates about examples

Suppose we have a labeled example in a dataset (x_1, y_1)

We can write this as a predicate: $\text{Label}(x_1, y_1)$

This is the statement "*The label for example* x_1 *is* y_1".

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A dataset is nothing but a collection of examples of the form $\{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$

This is equivalent to the conjunction

$$\bigwedge_{i=1}^{n} \text{Label}(x_i, y_i)$$

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Can we fold this into the framework?

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Let us fold them into one term

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(perhaps including properties about specific examples)

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Minimize Constraint loss

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Suppose we have a sentence α in predicate logic, defined over some atoms $X = \{X_1, X_2, \cdots, X_n\}$

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Suppose each atom X_i is associated with a probability p_i, possibly from a neural model

Let the vector $\mathbf{p}$ denote the collection of probabilities $[p_1, p_2, \cdots, p_n]$ over the atoms

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Our goal:

To define a loss function $L(\alpha, p)$ such that minimizing it produces a model (and associated probabilities) that assigns labels satisfying the sentence α

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- Often in practice, we will instantiate the constraint to a finite set of examples (possibly and often unlabeled)

Summary

The main idea:

- Convert constraints in symbolic logic to loss functions
- Minimize the logic-based loss
- Use any network architecture and any optimizer
- At inference time, there is no change in how we use our networks

  The hope: By minimizing constraint violations, our resulting models will also satisfy constraints in future examples

Coming up: Two instantiations of constraint loss

1. Semantic loss
2. T-norm losses