Neuro-Symbolic Modeling: Overview

Some examples of neural-symbolic integration

THE UNIVERSITY OF UTAH

This lecture

- The Two Systems of Thinking
- Learning & Reasoning
- History: Statistical relation learning
- Some examples of neural-symbolic integration
- Technical challenges for neural-symbolic integration
- A taxonomy of approaches

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2. Semantic role labeling

3. Using the rules of a game to play the game

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Digit prediction

Examples from the MNIST digit recognition dataset

The standard task

Given an image, recognize the digit in the image

Typically modeled as a multiclass classification task

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Label() = 5

Label( ) = 1

The label for the input  is 5

The label for the input is 1

These statements can be written as predicates

`Digit(`  `, 5)`

`Digit(` `, 1)`

Model decisions as predicates

“The model predicts a digit y for an input image x” = `Digit`(x, y)

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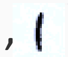
Sum( , ) = 6 The sum of these two digits is 6

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We can imagine another model that predicts the sum of two digit images

Sum( ,) = 6 The sum of these two digits is 6

As a predicate: `Sum` ( , , 6)

"The model predicts a sum y for input images x_1, x_2 " = `Sum` $(x_1,\ x_2,\ y)$

Digits and sums

The `Digit` and `Sum` models can be both implemented as CNNs

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`If` The image x_1 has the digit z_1 `and` The image x_2 has the digit z_2

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$$\texttt{Digit}(x_1,\ z_1) \wedge \texttt{Digit}(x_2,\ z_2) \wedge \boxed{(y = z_1 + z_2)} \rightarrow \texttt{Sum}(x_1,\ x_2,\ y)$$

`If` The image x_1 has the digit z_1 `and` The image x_2 has the digit z_2 `and` The numbers z_1 and z_2 add up to y

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Given data for one task and this rule, can we train a model for the other task?

Let’s look at a few examples

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Semantic Role Labeling (SRL)

Who did what to whom, where, when, why?

The mouse ate the cheese when the farmer left the kitchen.

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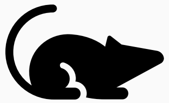
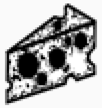
Arg0 Arg1 ArgM-TMP

	ate
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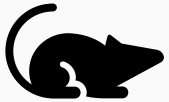
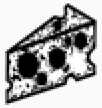
the farmer	left	the kitchen
Arg0		Arg1





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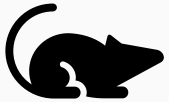
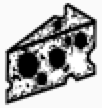
These *semantic roles* are defined by the PropBank data (Palmer et al)





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What do the labels mean?

The mouse ate the cheese when the farmer left the kitchen.

ate		left	
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PropBank defines what the labels `Arg0`, `Arg0` etc mean for each verb

What do the labels mean?

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Agent: eater, consumer

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Temporal *modifier*

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Agent: entity in motion

Starting point, location vacated

PropBank defines what the labels `Arg0`, `Arg0` etc mean for each verb

Semantic Role Labeling: The modeling problem

- Input: A sentence
- Output: Semantic frames for all verbs

A well studied task, large datasets in English

- Penn Treebank data, Ontonotes annotated with PropBank roles
- Creating datasets is not easy though

Semantic Role Labeling: The modeling problem

- **Input**: A sentence
- **Output**: Semantic frames for all verbs

The output is *structured*, and has constraints about the labels. For example

- Core arguments (e.g. `Arg0`, `Arg1`) cannot repeat…
  …but modifiers (e.g. `ArgM-TMP`) can

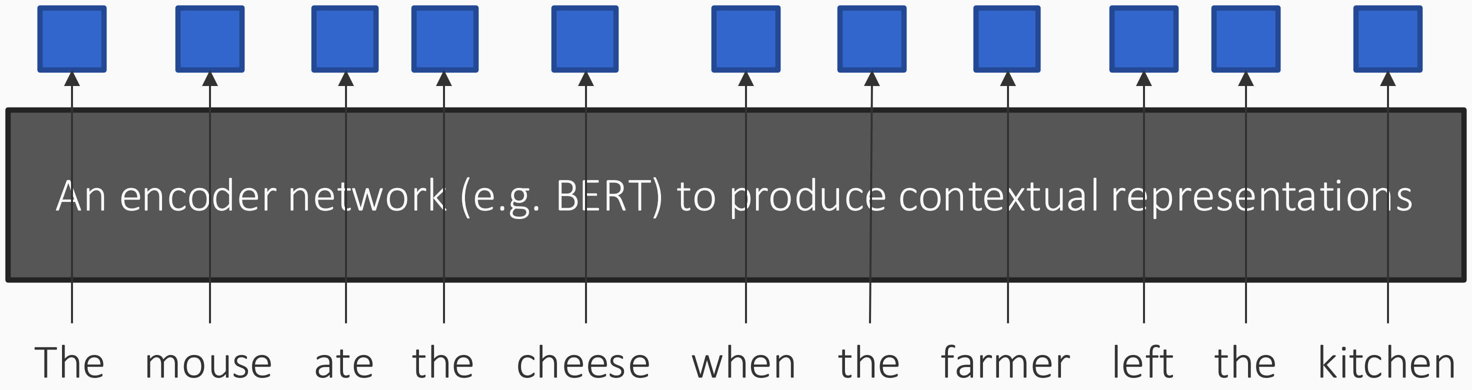
- Certain arguments (called references, e.g. `R-Arg0`) can appear only if the corresponding referent argument exists (here, `Arg0`)

These symbolic constraints come from the task definition. And linguistic assumptions.
We will see other constraints later

A common design of neural SRL models

The mouse ate the cheese when the farmer left the kitchen

A common design of neural SRL models



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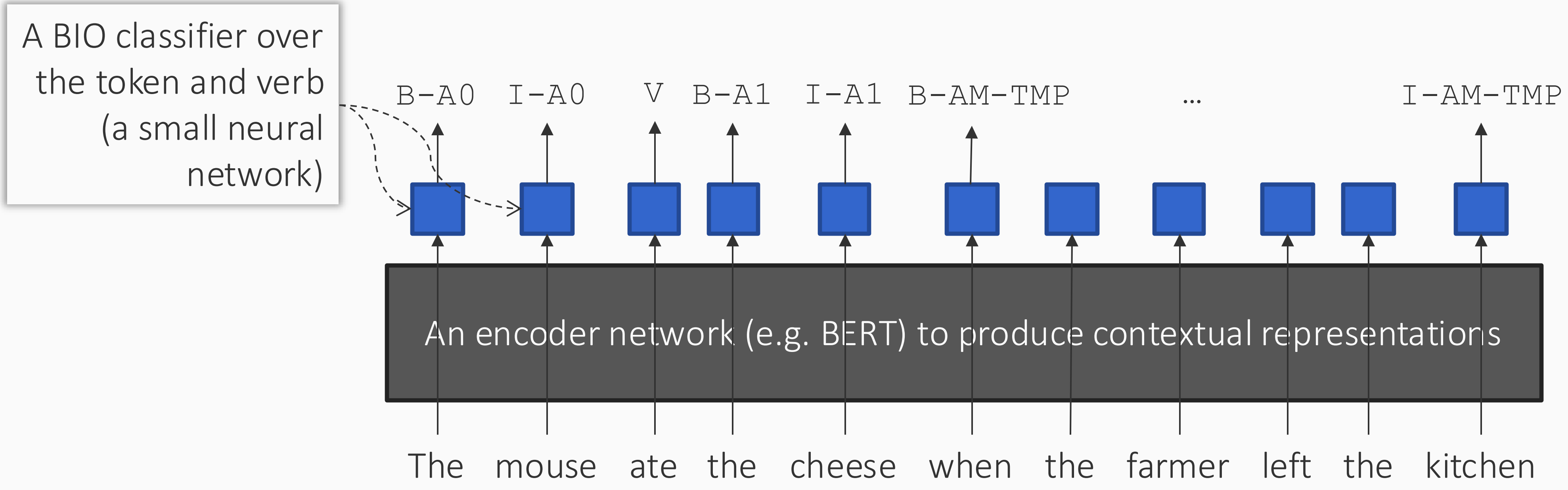

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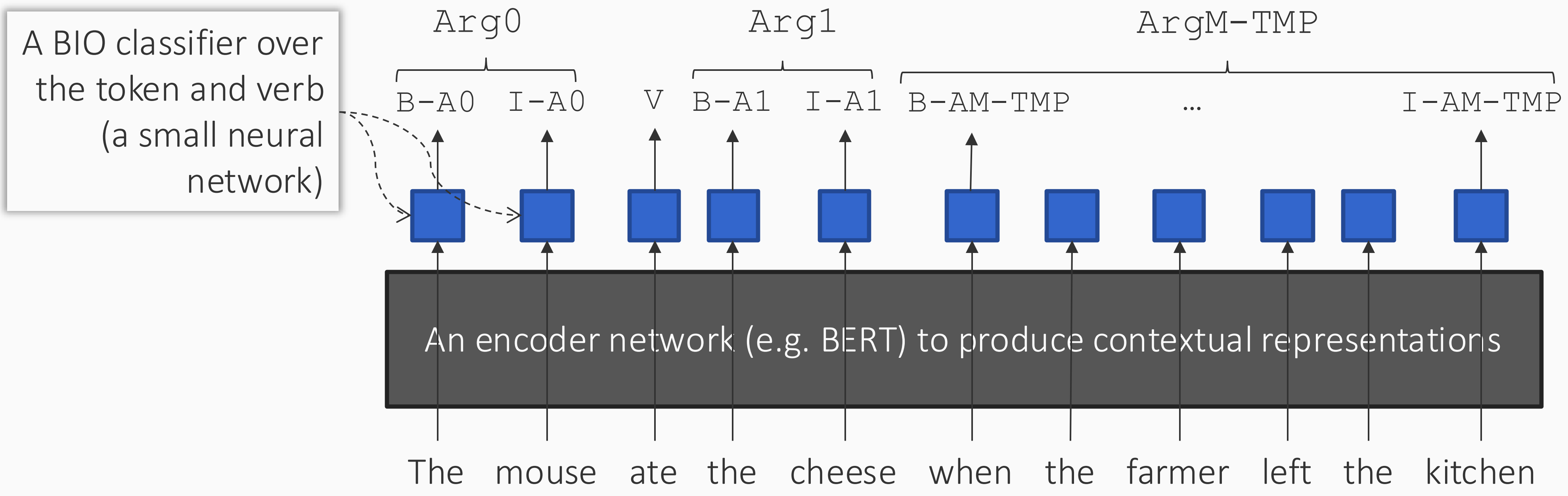

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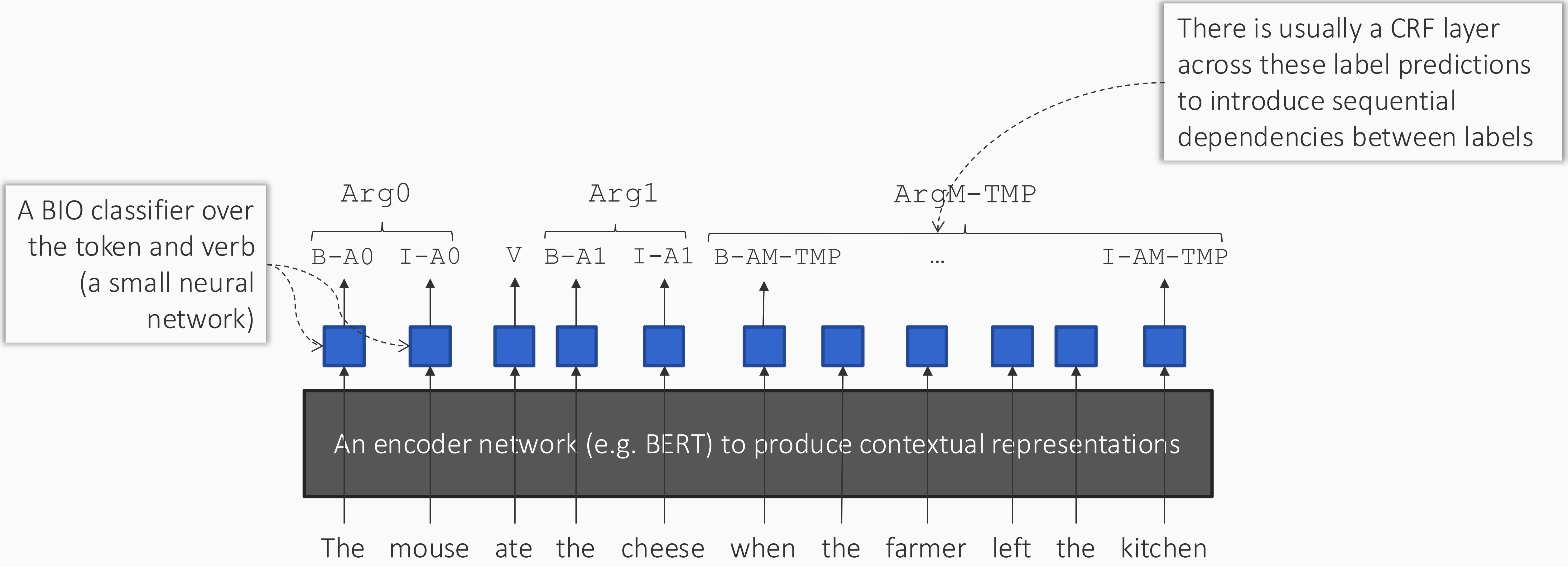
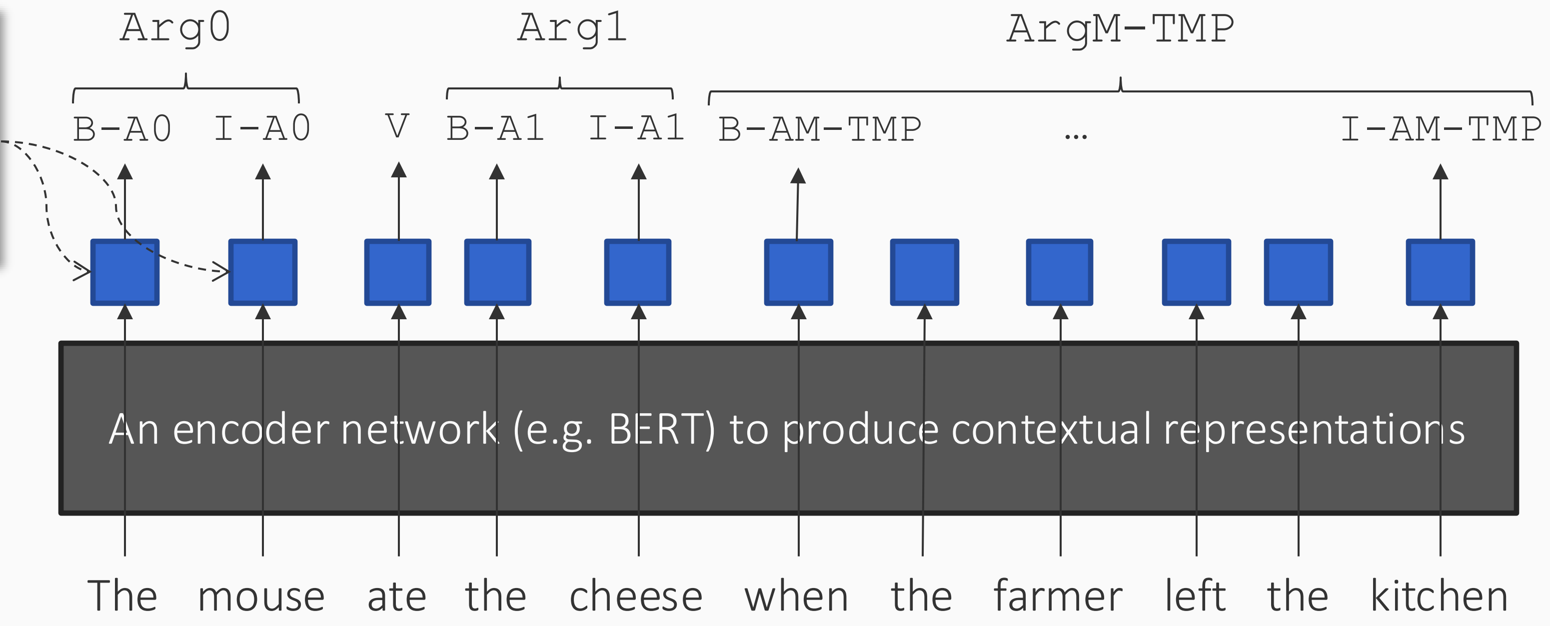


A common design of neural SRL models

A BIO classifier over the token and verb (a small neural network)



Each predicate treated separately. Here we see the predictions for the verb *ate*.

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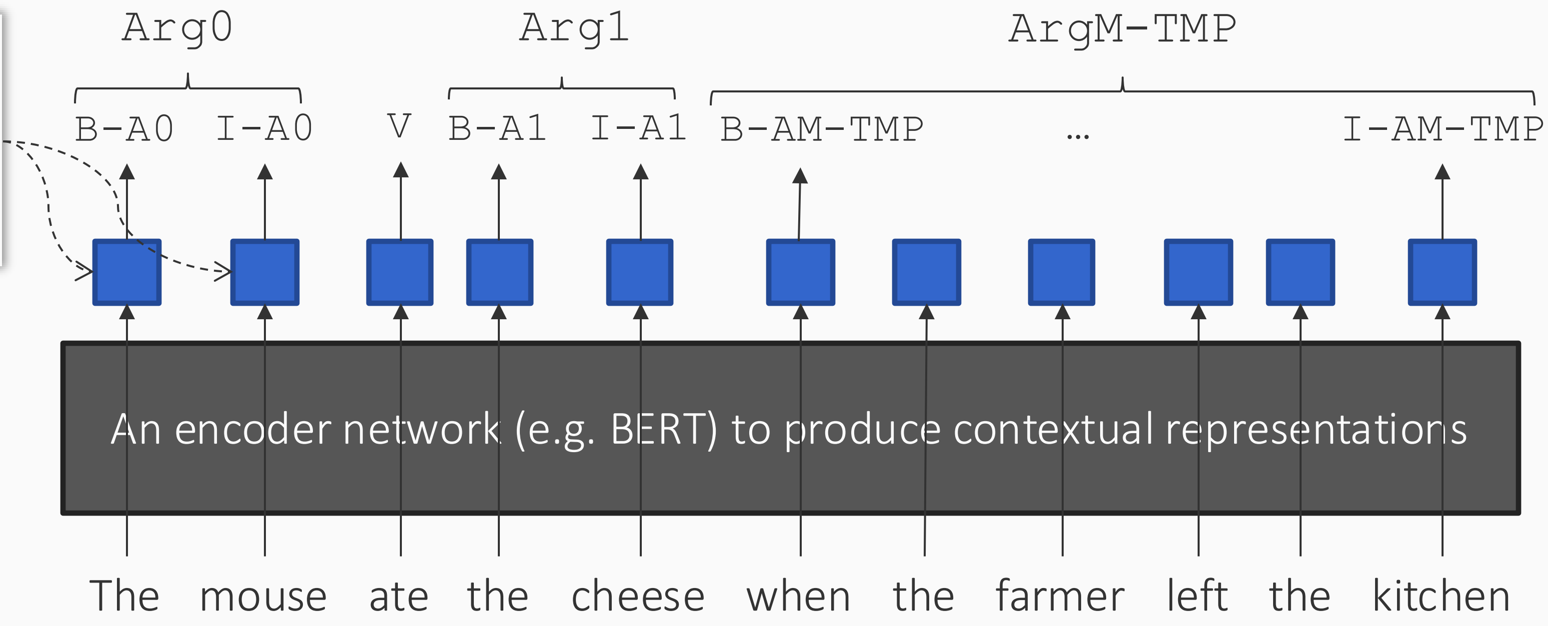
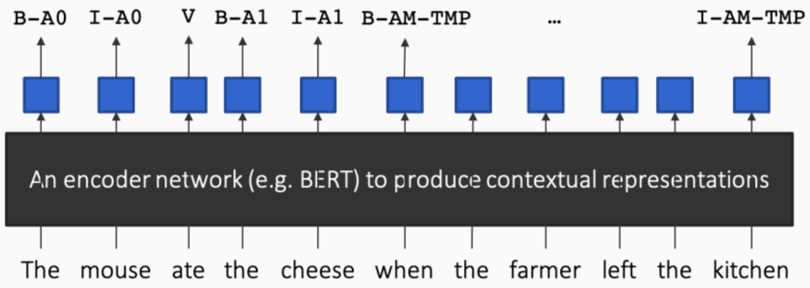


A different commonly seen design involves span-based predictions instead of word-level ones.

A remarkable class of models!



These models are excellent in terms of predictive accuracy

- E.g. with RoBERTa embeddings, on Wall Street Journal data, we can get ~88% F-scores
- And ~80% on out of domain (Brown corpus) sentences

Impressively, they do so with no information about the output space

- No symbolic constraints prohibiting invalid labels
- No structural information about references, etc
- No frame information

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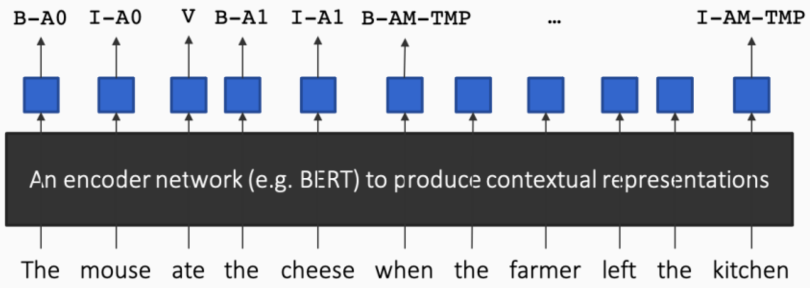

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Is there room for knowledge in the form of symbolic constraints in neural SRL models?

- If so, how do we introduce it without sacrificing modeling convenience?

Constraints in SRL: Unique Core Roles

For any verb u, and a word i

$$\forall\, u, i \in s$$

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The model cannot predict that it is the beginning of the same label

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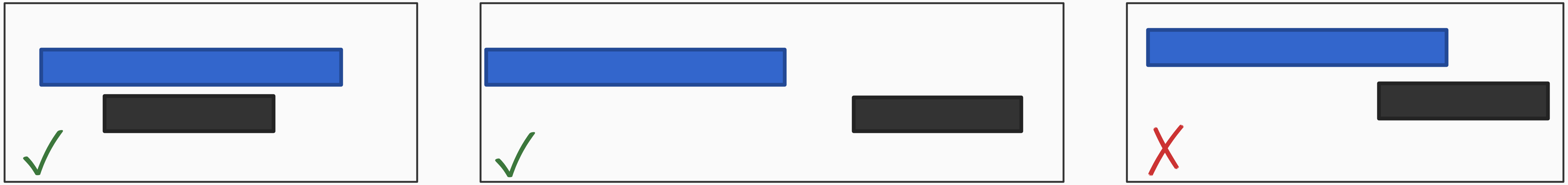
Other constraints (informally)

The exclusively overlapping role constraint:

– In any sentence, an argument for a predicate can either be contained in, or fully outside, the argument for any predicate



The frame core role constraint

– A verb can have only those core arguments that are defined in PropBank

The modeling questions

How can we incorporate such knowledge into our modeling and/or prediction process?

- The constraints are not dependent on any specific labeled example
- The labeled data presumably already satisfies the constraints

Can such knowledge help?

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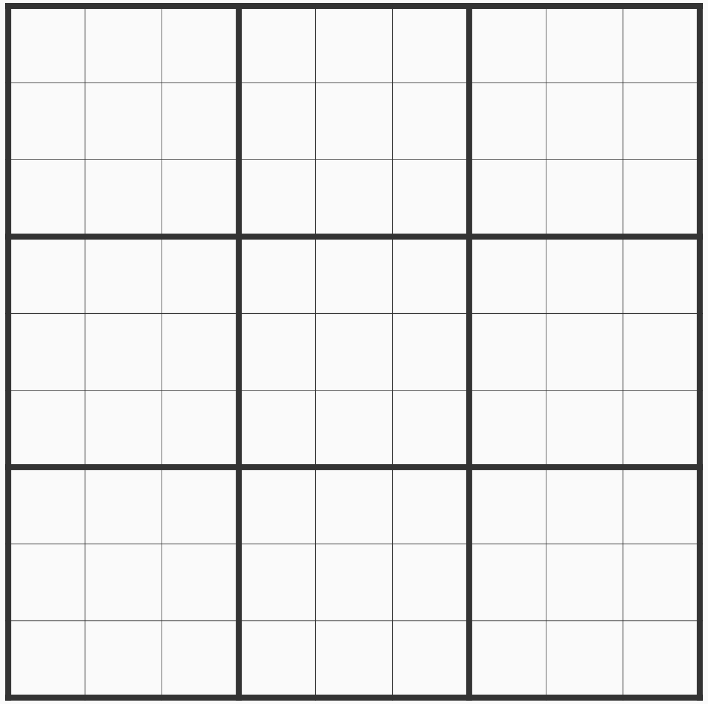
Can we instruct a model to play a game by giving it the rules of the game?



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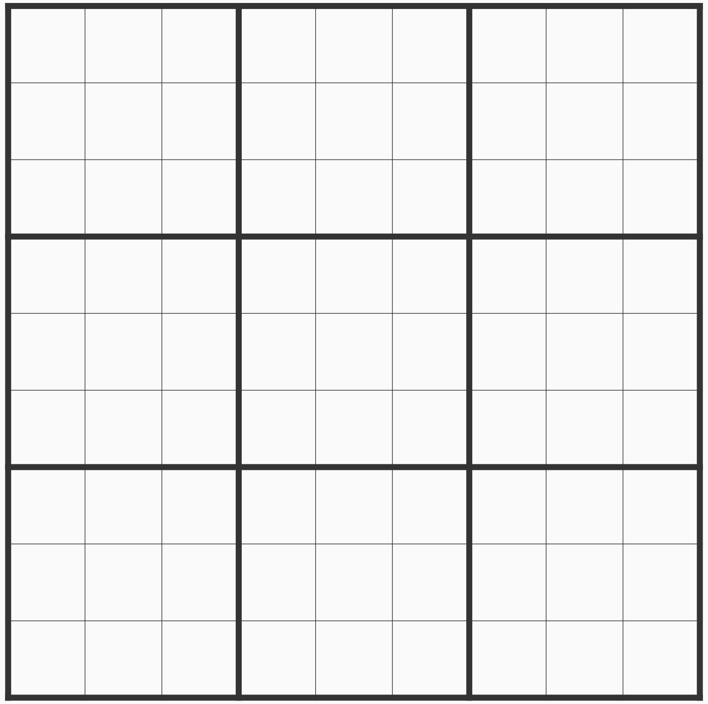

Example: sudoku

1. Each cell can be filled a digit from 1 to 9

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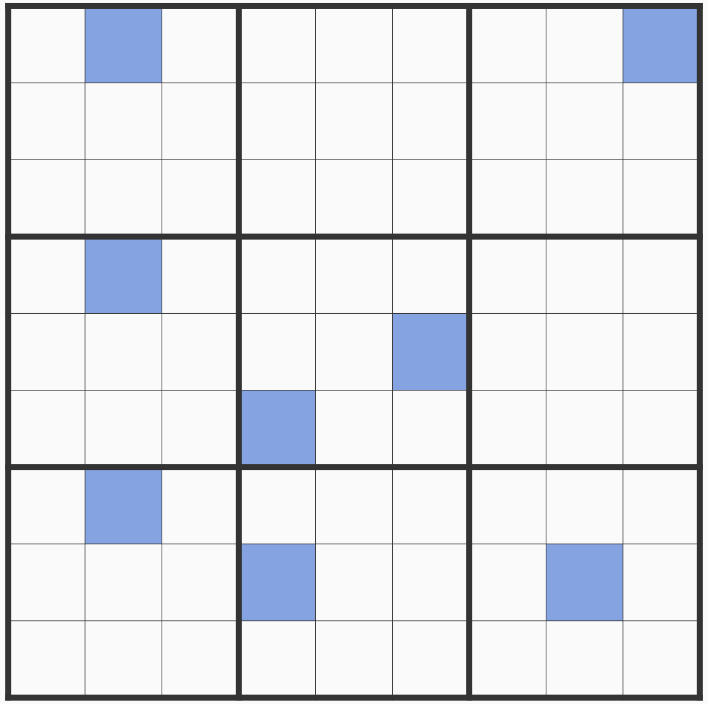

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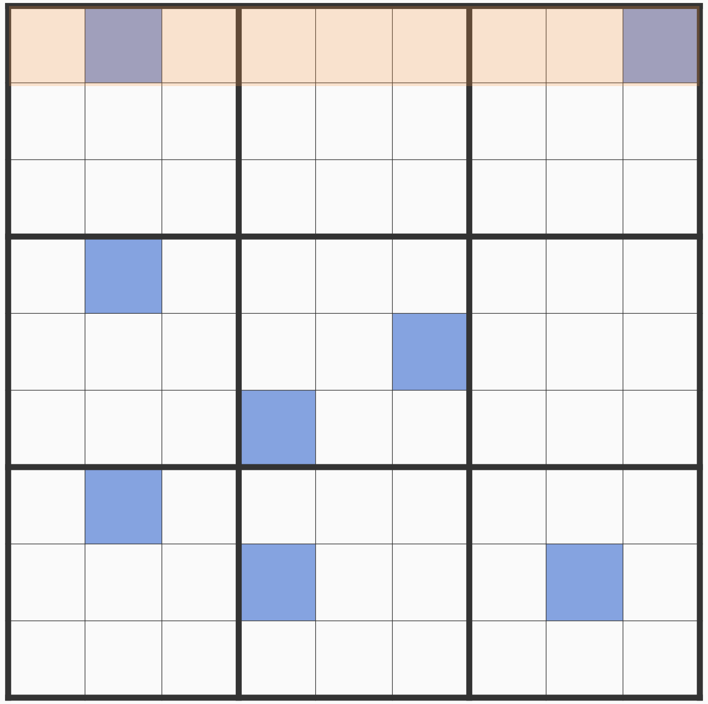

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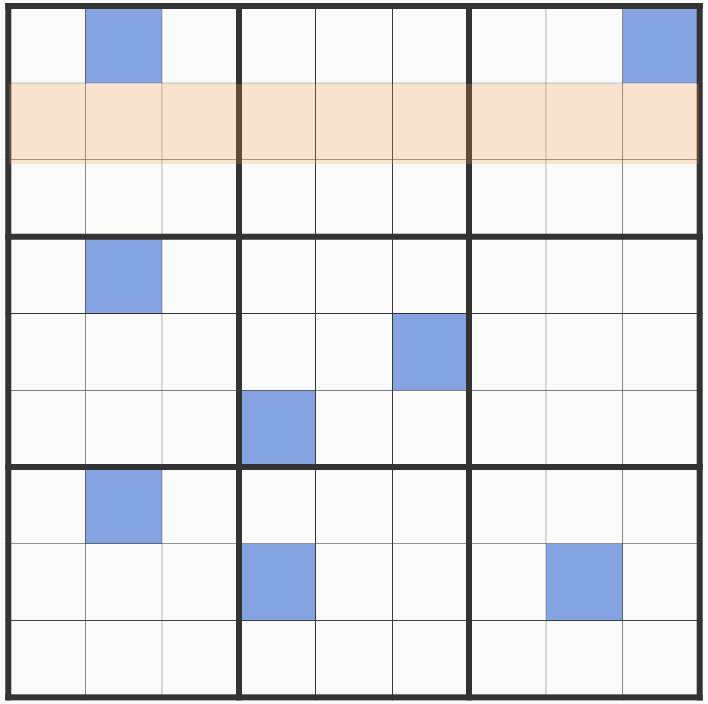

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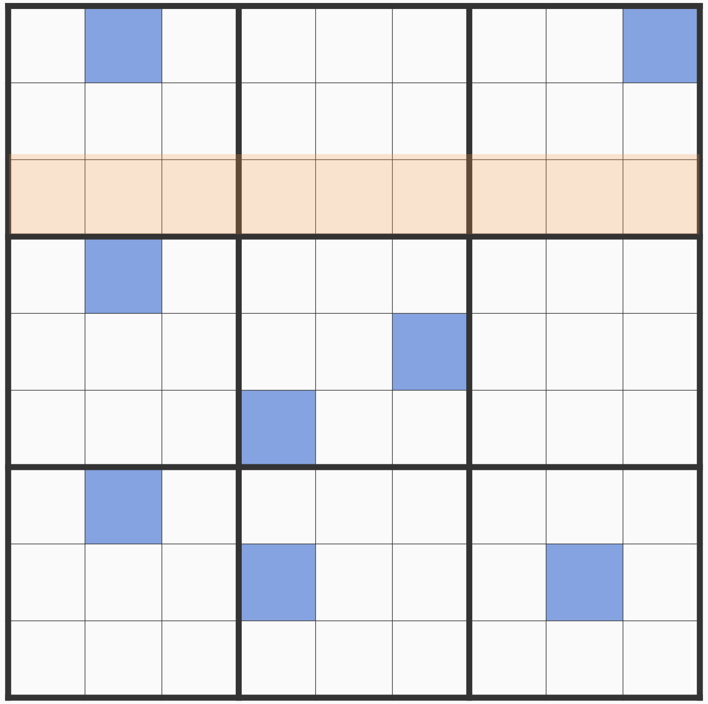

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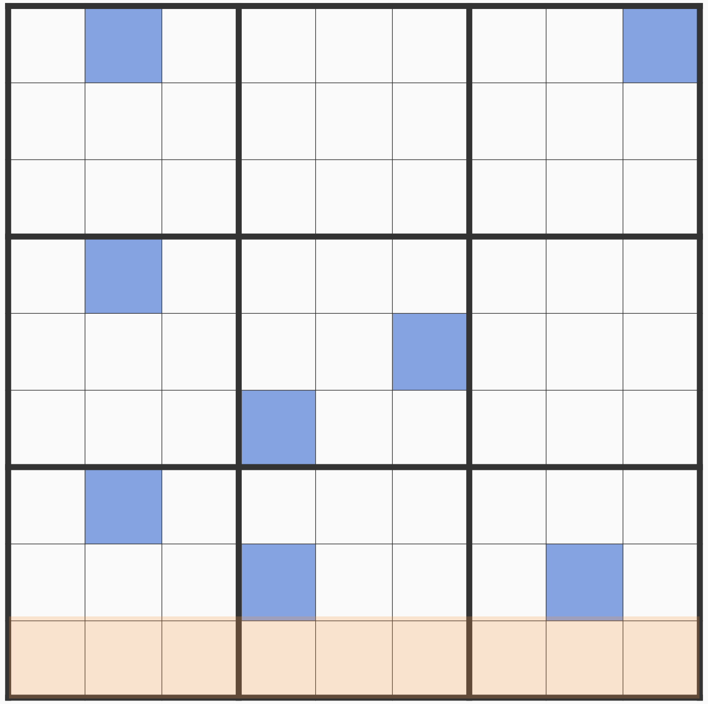

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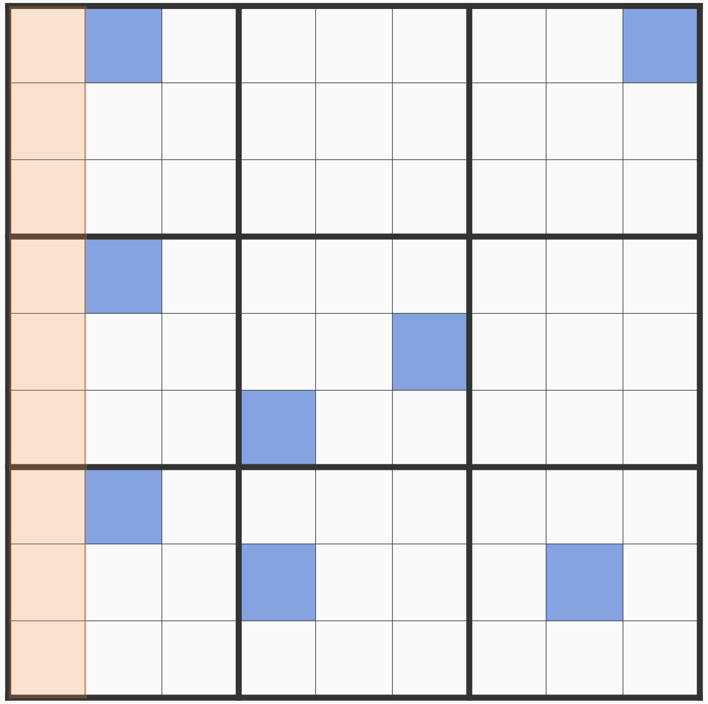

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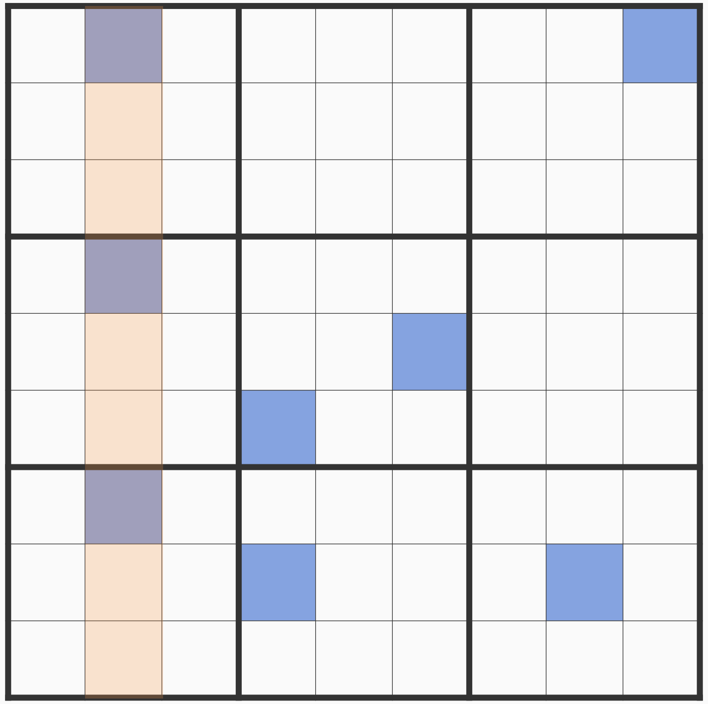

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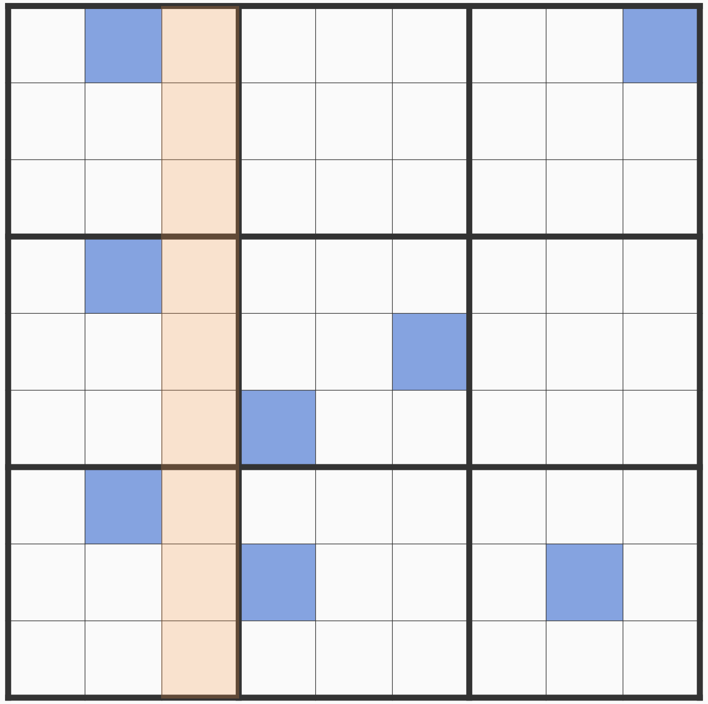

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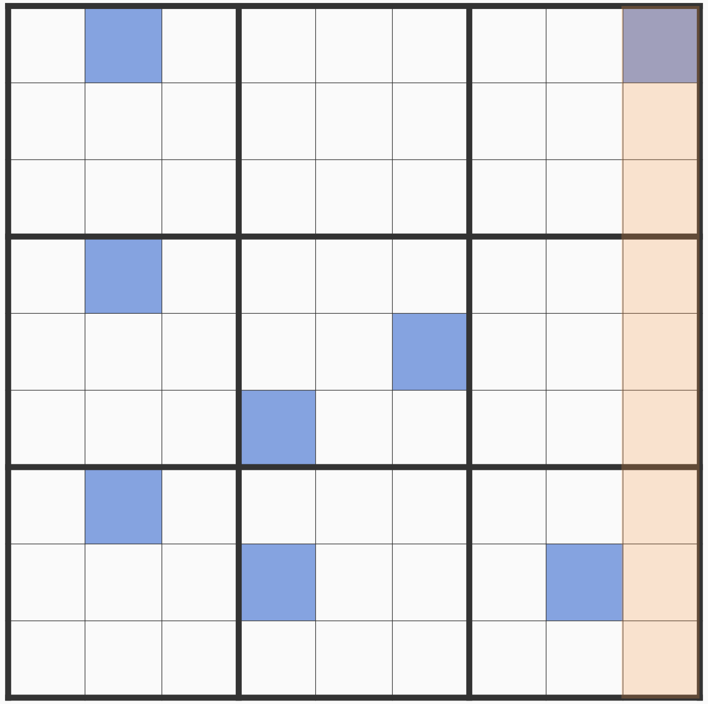

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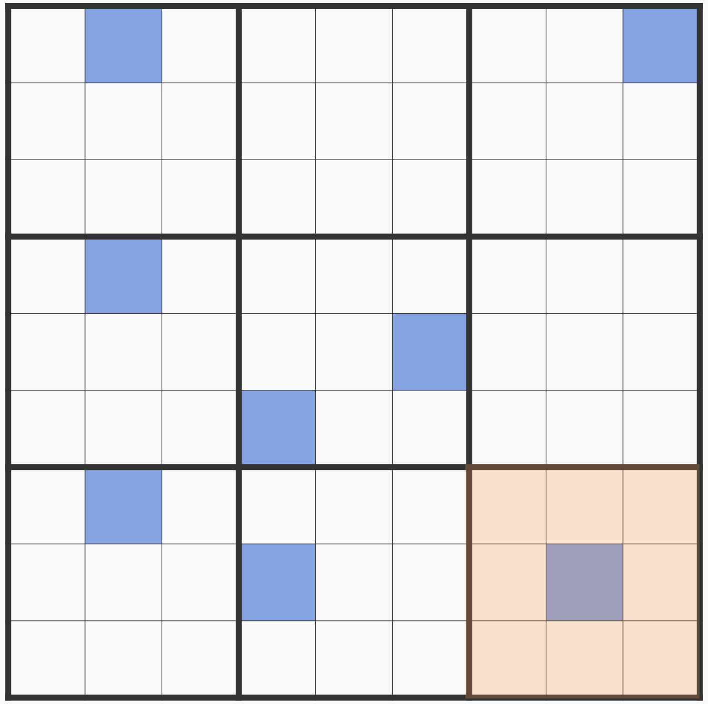

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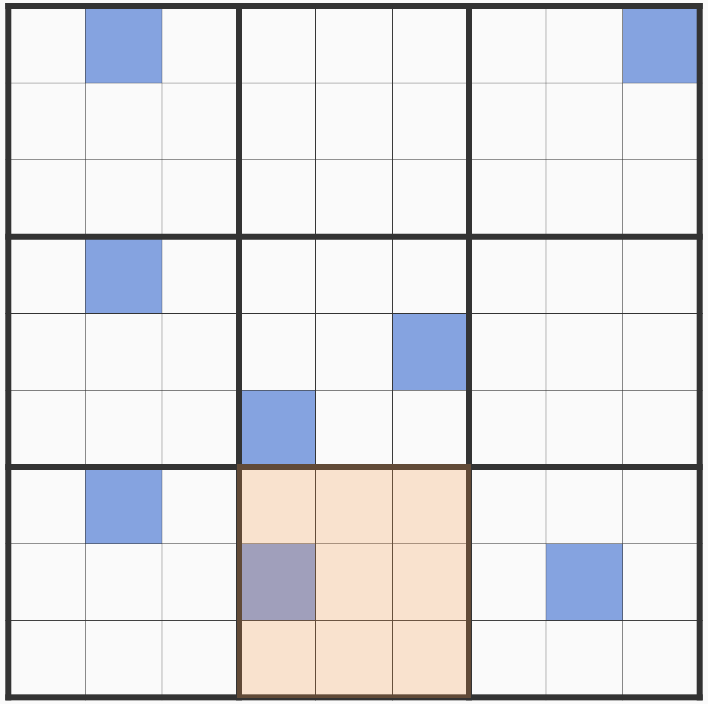

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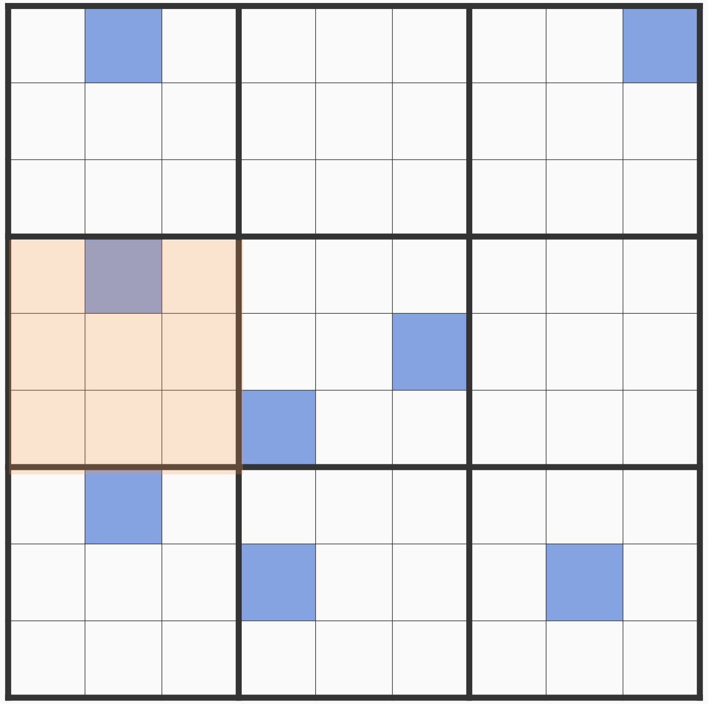

Example: sudoku

1. Each cell can be filled a digit from 1 to 9
2. Some cells are already filled in
3. All digits from 1-9 should show up exactly once in each row
4. All digits from 1-9 should show up exactly once in each column
5. All digits from 1-9 should show up exactly once in each 3x3 block

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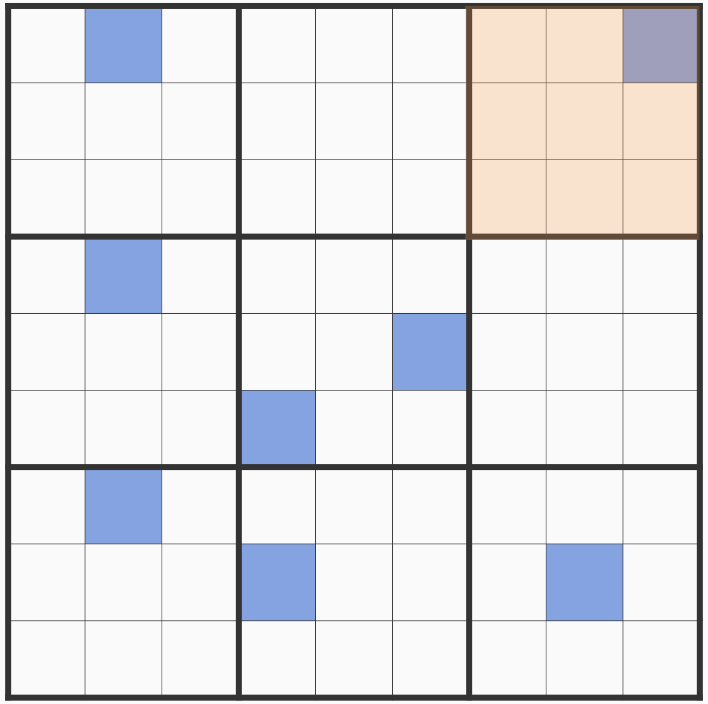

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We have nine blocks in all

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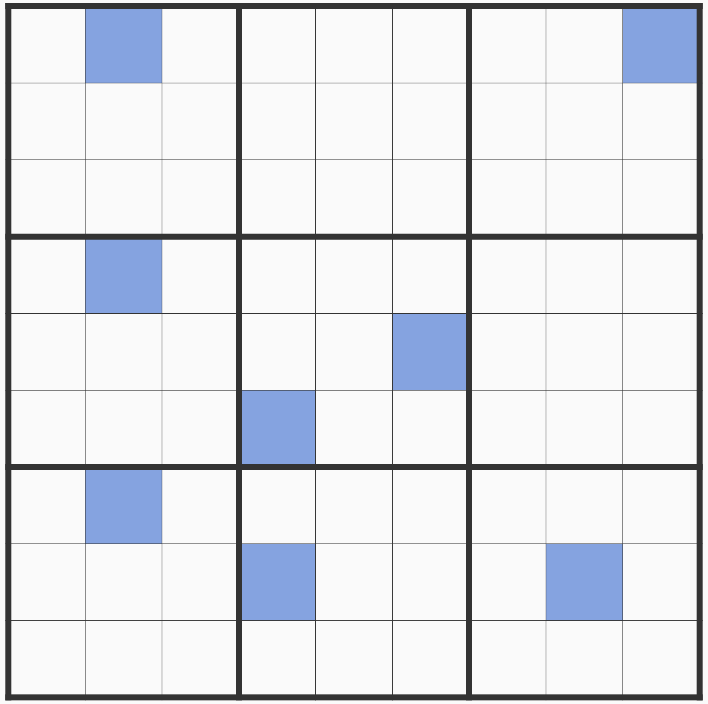

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Note: Sudoku is an example of a constraint satisfaction problem. We don't need to train a neural network to solve the problem.

For the purpose of this course, we are asking these questions to demonstrate what *can* be done, not necessarily what *should* be done

Let’s look at a few examples

1. Recognizing digits and adding them

2. Semantic role labeling

3. Using the rules of a game to play the game

4. Other examples

Other motivating examples

Can we construct a knowledge base using facts learned from the internet and also knowledge about the world such as "The father of a father is a grandfather"?

Can we write a program that operates over multiple independent large language model predictions to produce an output that is consistent with some pre-defined rules?

Can we train a neural network with only a limited amount of training data for a task, given that we have some knowledge about the task?