Logic as Loss: Semantic loss

THE UNIVERSITY OF UTAH

The idea of the "logic as loss" framework

What we want of our models

Satisfy a set of invariant properties
(perhaps including properties about specific examples)

Minimize Constraint loss

Let us formally state the setting

Suppose we have a sentence α in predicate logic, defined over some atoms $X = \{X_1, X_2, \cdots, X_n\}$

Suppose each atom X_i is associated with a probability p_i, possibly from a neural model

Let the vector $\mathbf{p}$ denote the collection of probabilities $[p_1, p_2, \cdots, p_n]$ over the atoms

Our goal:

To define a loss function $L(\alpha, \mathbf{p})$ such that minimizing it produces a model (and associated probabilities) that assigns labels satisfying the sentence α

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What are some desirable properties of the loss function $L(\alpha, p)$?

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Logic as loss: Semantic loss

- Building up to semantic loss: The axioms
- Semantic loss
- Examples
  - Conjunction
  - Implication
- Complex constraints & Weighted Model Counting
  - Knowledge Compilation
  - Example: The exactly-one constraint

A second look at the semantic loss

$$L(\alpha, \mathbf{p}) \propto -\log \sum_{x \models \alpha} \left(\prod_{i : x \models X_i} p_i \cdot \prod_{i : x \models \neg X_i} (1 - p_i) \right)$$

Is there a catch?

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Is this a problem? Have we seen this before?

Weighted model counting

Recall: model = a satisfying assignment to a propositional formula

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Model counting: How many satisfying assignments does the propositional formula have?

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X_1	X_2	α
$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\top$
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Not a model for this formula

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X_1	X_2	α
$\top$	$\top$	$\top$
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$\bot$	$\top$	$\top$
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Number of models = 3

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Variable weights

X	$w(X)$	$w(\neg X)$
X_1	3	2
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Example: $X_1 \rightarrow X_2$

X_1	X_2	α	weight
⊤	⊤	⊤	$3 \times 5 = 15$
~~⊤~~	~~⊥~~	~~⊥~~	~~$3 \times 7 = 21$~~
⊥	⊤	⊤	$2 \times 5 = 10$
⊥	⊥	⊤	$2 \times 7 = 14$

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Weighted model count = 15 + 10 + 14 = 39

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- Also #P

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Computing the semantic loss requires us to perform weighted model counting

Intractable in the worst case, *but tractable subsets of logic exist*

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One way to count:

Enumerate all satisfying assignments

X_1	X_2	X_3	α
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$\top$	$\top$	$\bot$	$\top$
$\top$	$\bot$	$\top$	$\top$
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Can we do better?

In the worst case, not really

X_1	X_2	X_3	α
$\top$	$\top$	$\top$	$\perp$
$\top$	$\top$	$\perp$	$\top$
$\top$	$\perp$	$\top$	$\top$
$\top$	$\perp$	$\perp$	$\perp$
$\perp$	$\top$	$\top$	$\perp$
$\perp$	$\top$	$\perp$	$\perp$
$\perp$	$\perp$	$\top$	$\perp$
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X_1	X_2	X_3	α
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One way to count:

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But are there easy cases?

An equivalent expression

$\alpha = X_1 \wedge \neg(X_2 \leftrightarrow X_3)$ is the same as $X_1 \wedge \big((X_2 \wedge \neg X_3) \vee (\neg X_2 \wedge X_3)\big)$

We can write this as an expression tree

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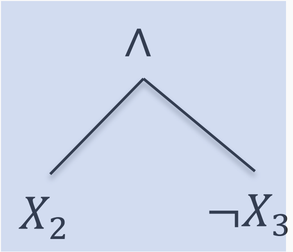

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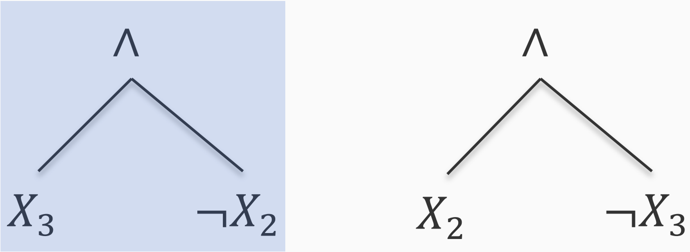

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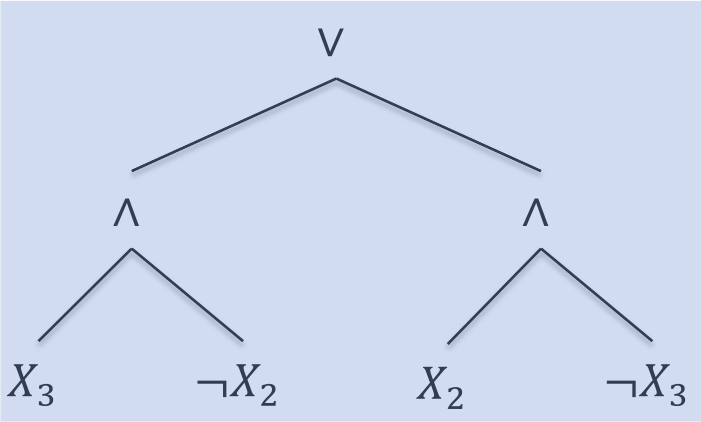

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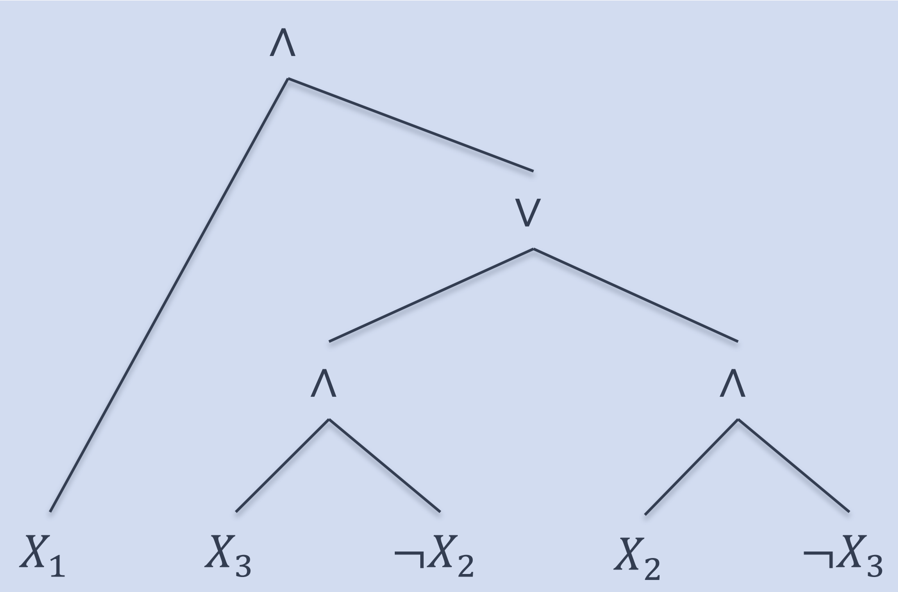

This expression is in the *Negation Normal Form*

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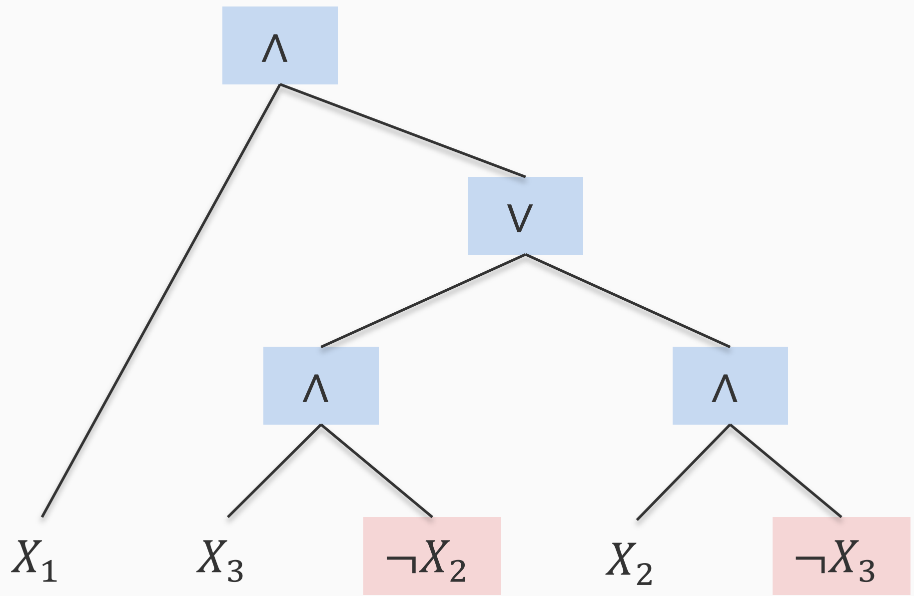

The only negations are in the leaves

That is, the leaves are literals

All other nodes are either conjunctions or disjunctions

This expression is *Decomposable*

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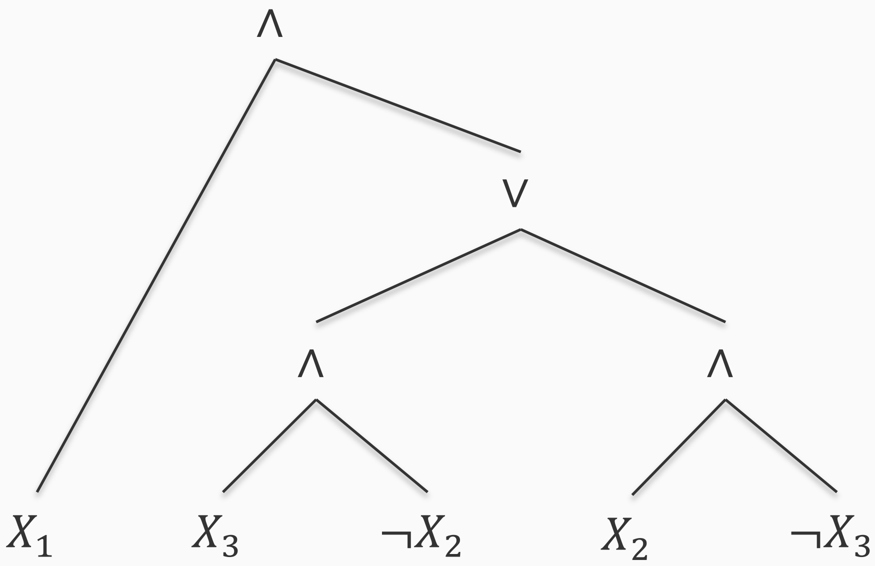
✓ Negation Normal Form



For every conjunction, the conjuncts do not share any variables

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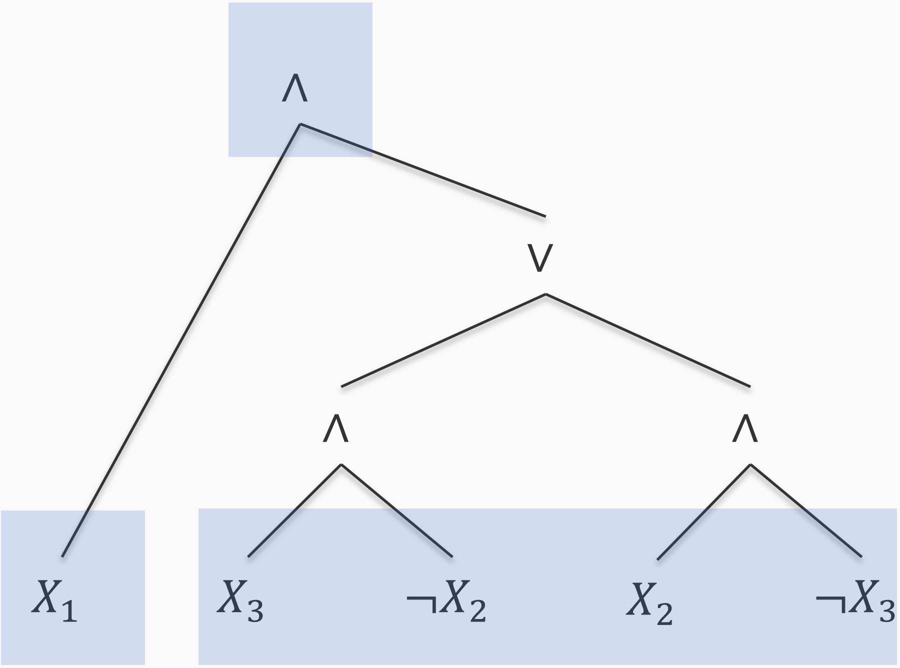

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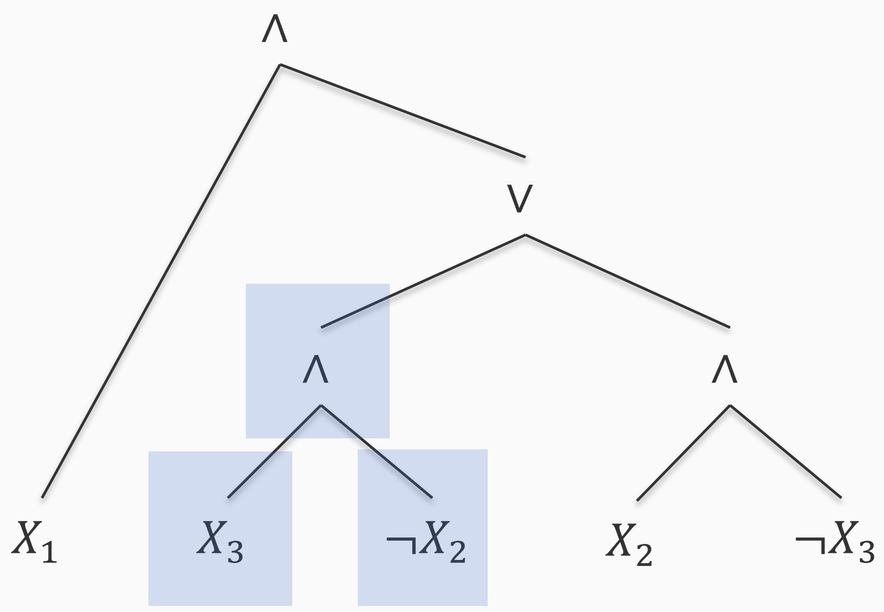

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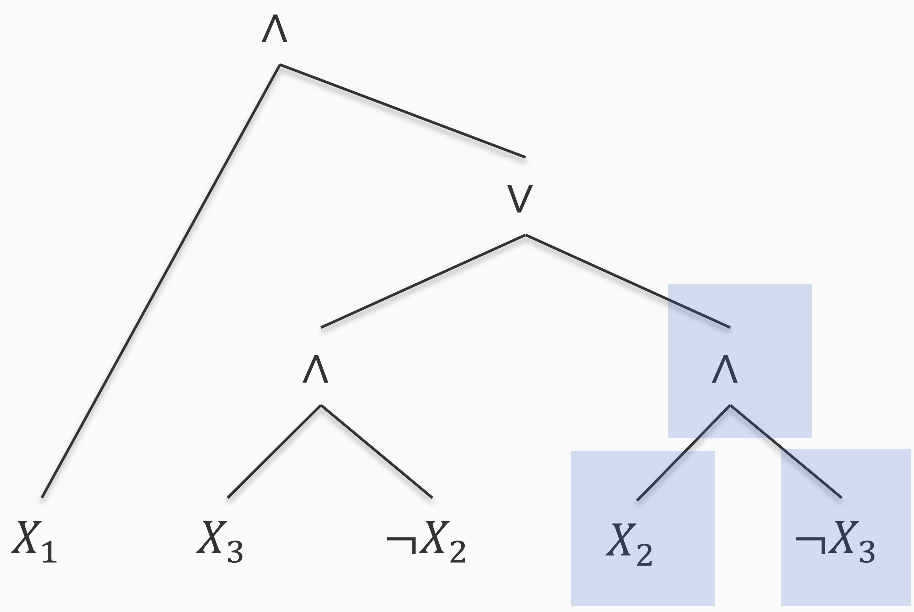

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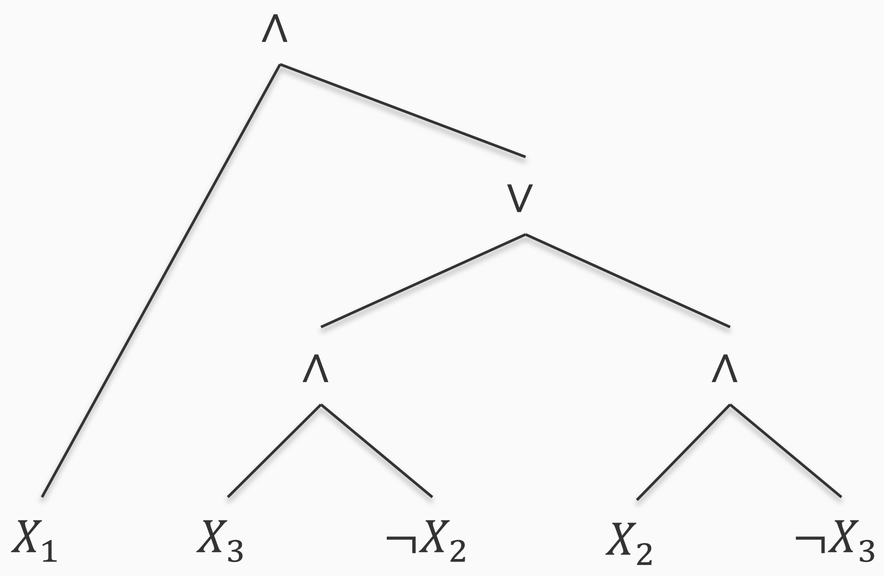

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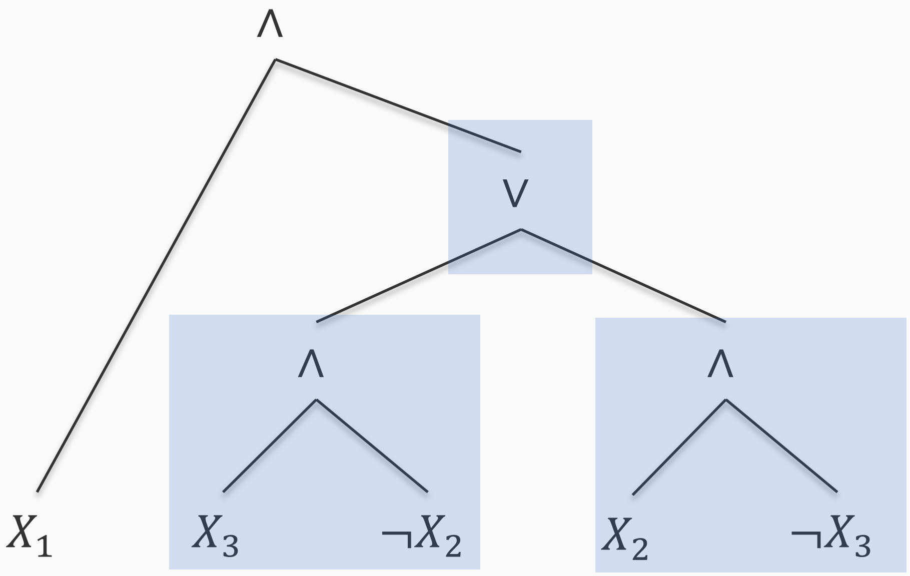

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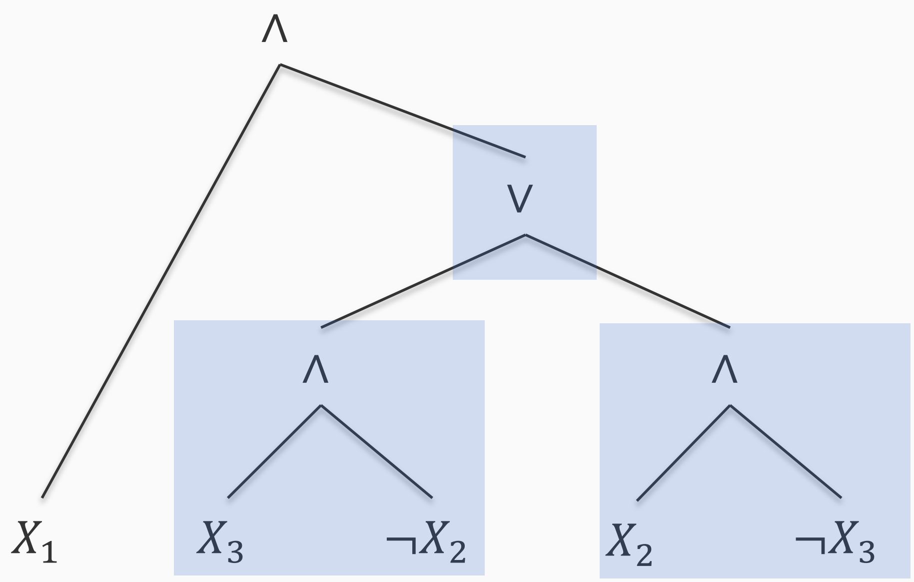

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The left side of the disjunction is $X_2 \land \neg X_3$

The right side is $\neg X_2 \land X_3$

Both these expressions cannot be simultaneously true
(you should verify this)

We have a new normal form

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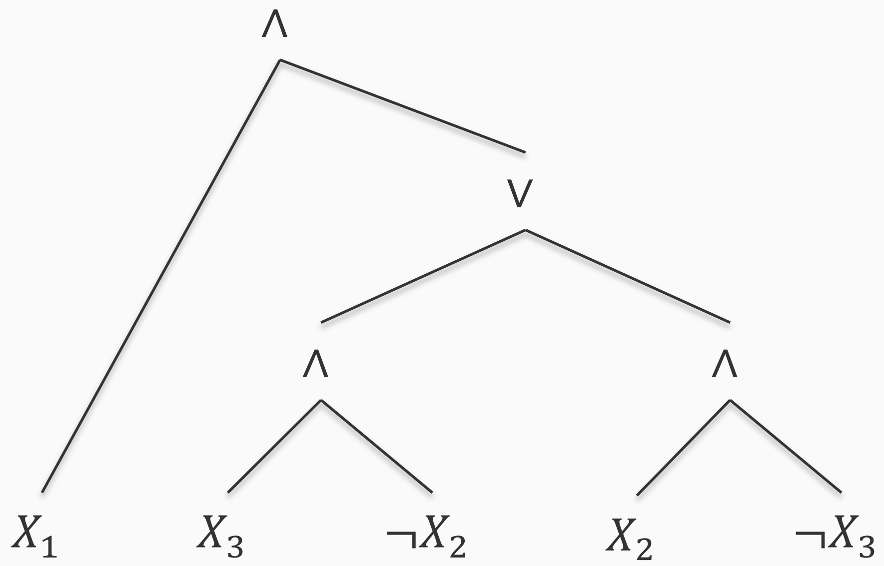

With all these three properties, this expression is in a form called deterministic decomposable negation normal form (d-DNNF).

We have seen normal forms of propositional formulas before

Disjunctive normal forms,

Conjunctive normal forms

Darwiche, Adnan, and Pierre Marquis. "A knowledge compilation map." *Journal of Artificial Intelligence Research* 17 (2002)

Darwiche, Adnan. "On the tractable counting of theory models and its application to truth maintenance and belief revision." *Journal of Applied Non-Classical Logics* 11, no. 1-2 (2001): 11-34.

What makes a d-DNNF special?

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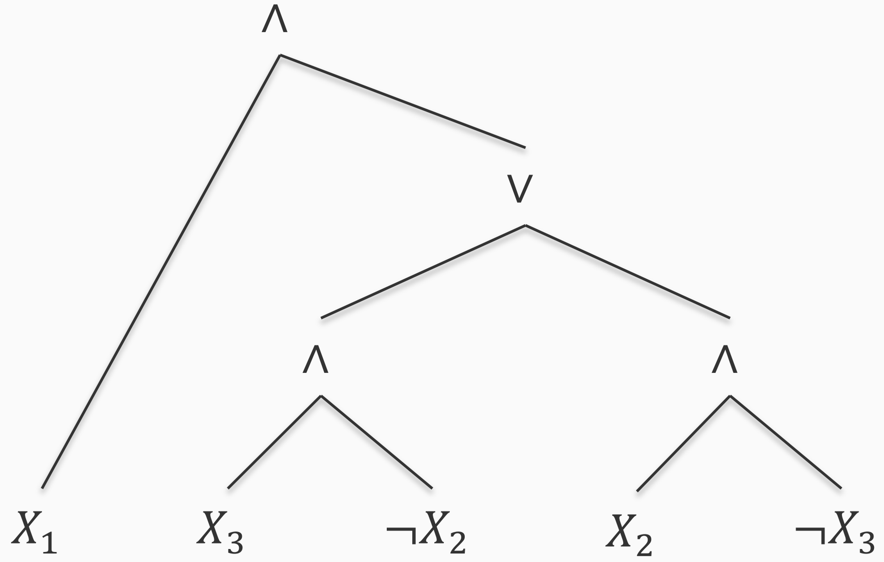

It allows us to perform model counting and weighted model counting with one traversal of the tree

Let’s see how

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Recall that the original expression
had two satisfying assignments

- ✓ Negation Normal Form
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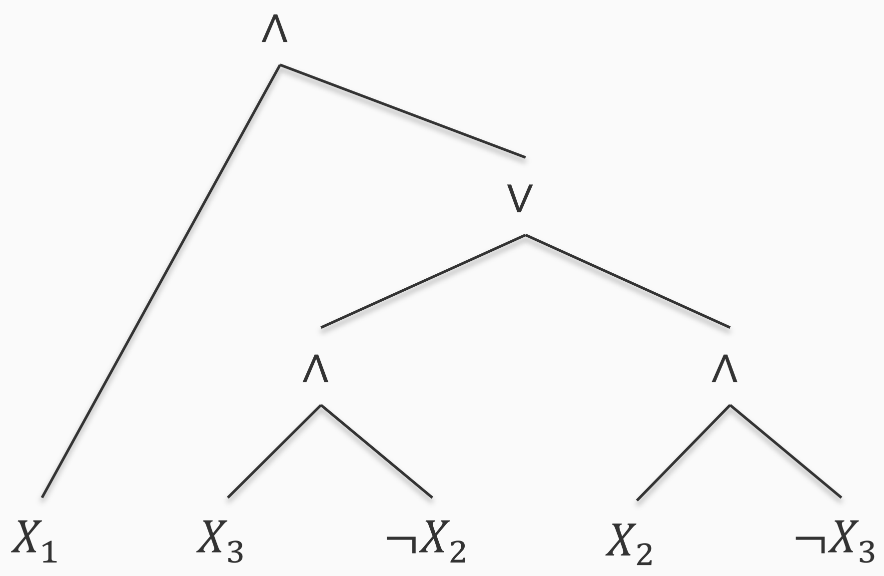

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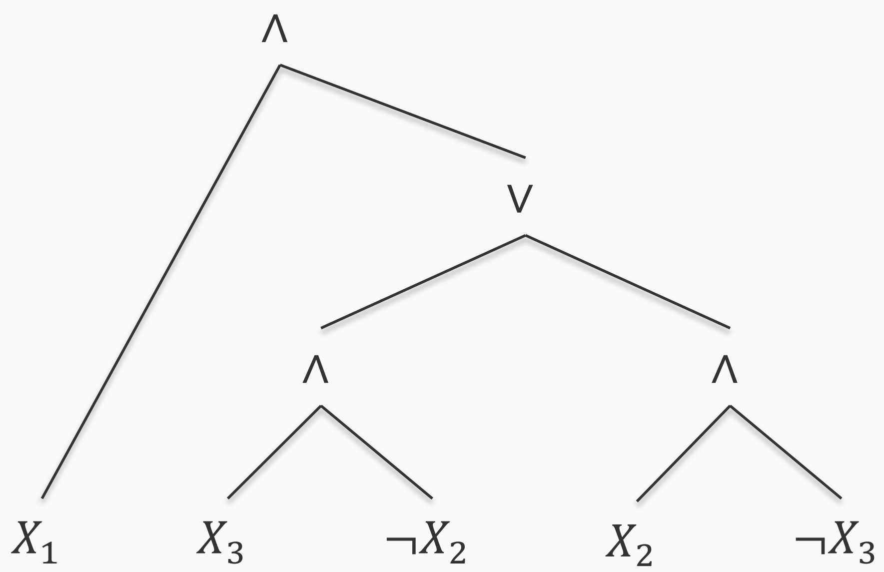

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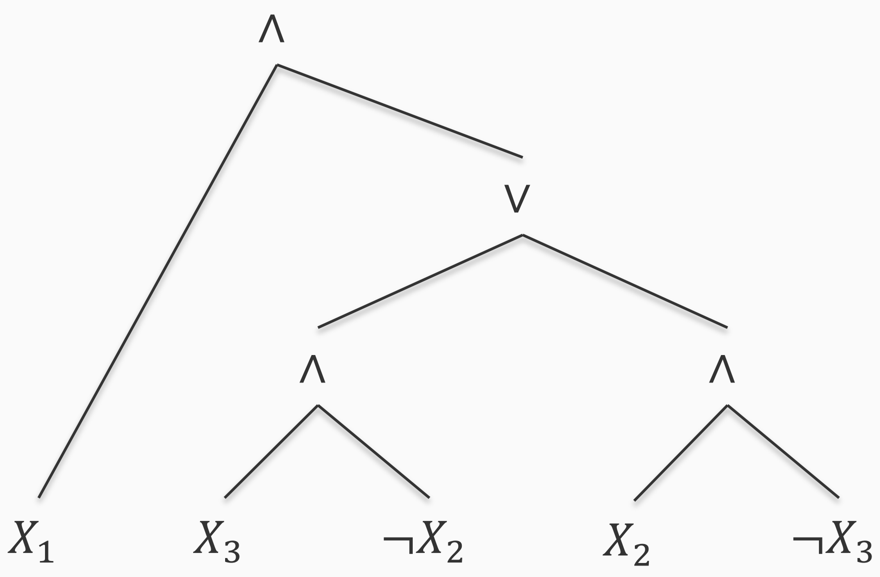

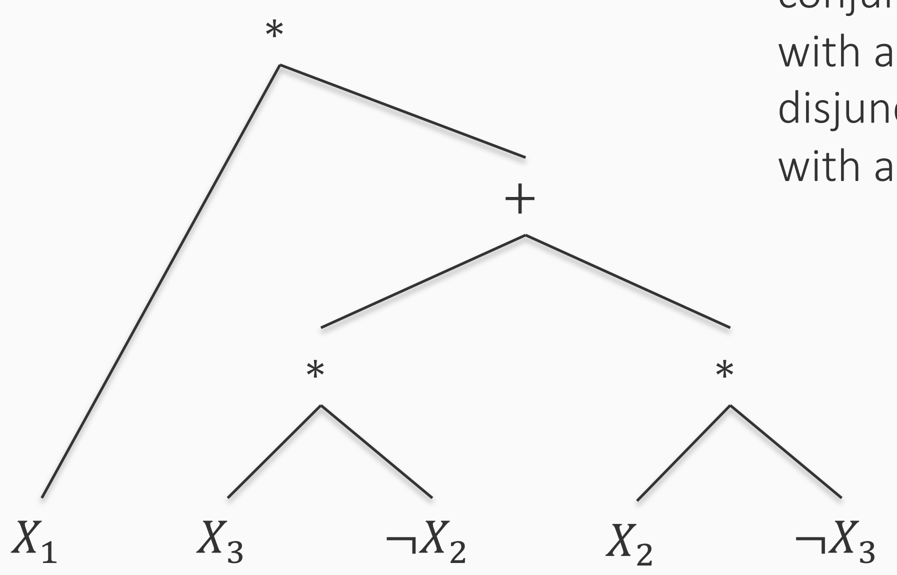

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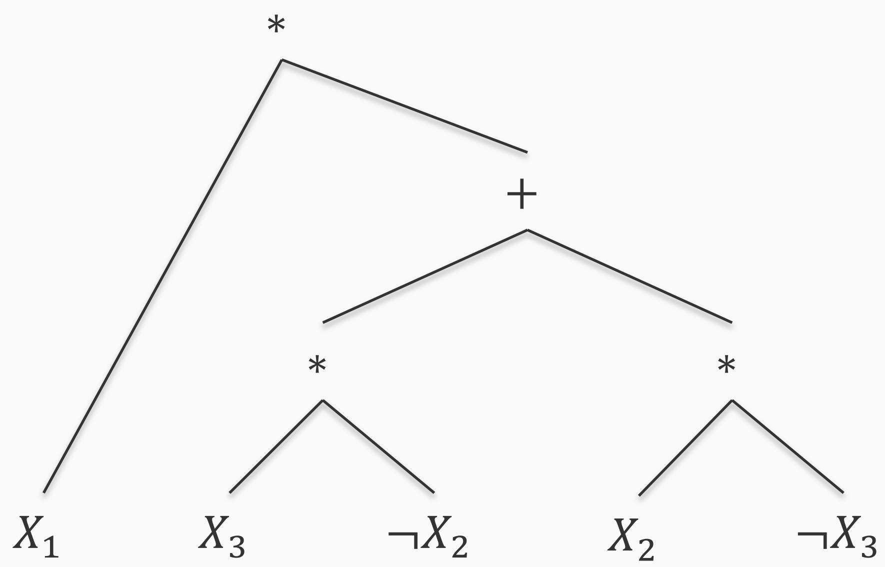
In the counting tree, assign every leaf node to take the value one…



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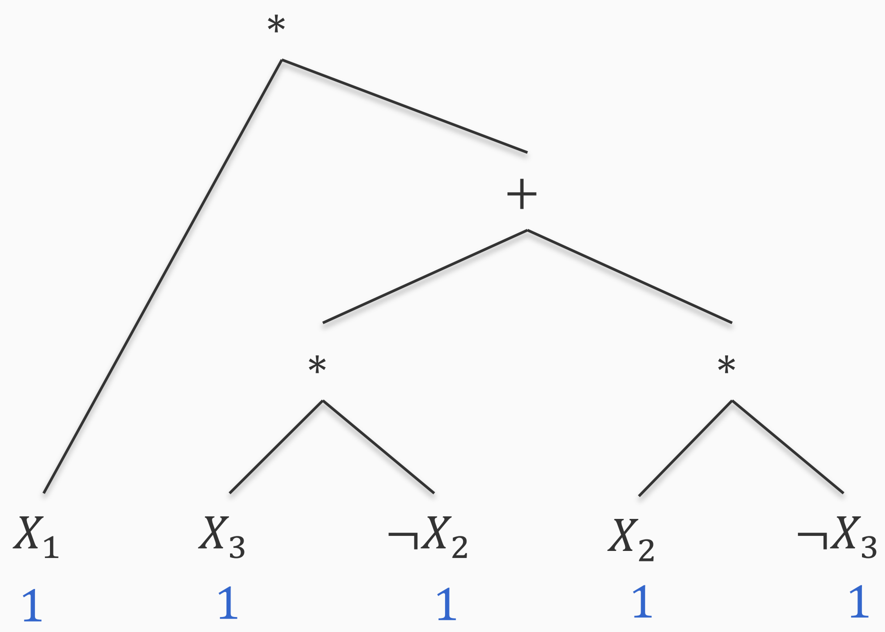

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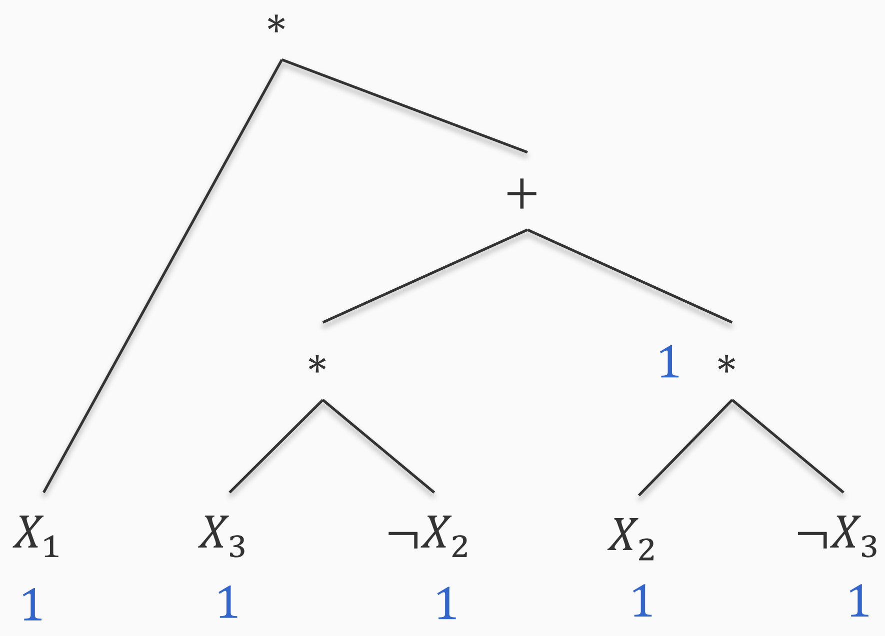

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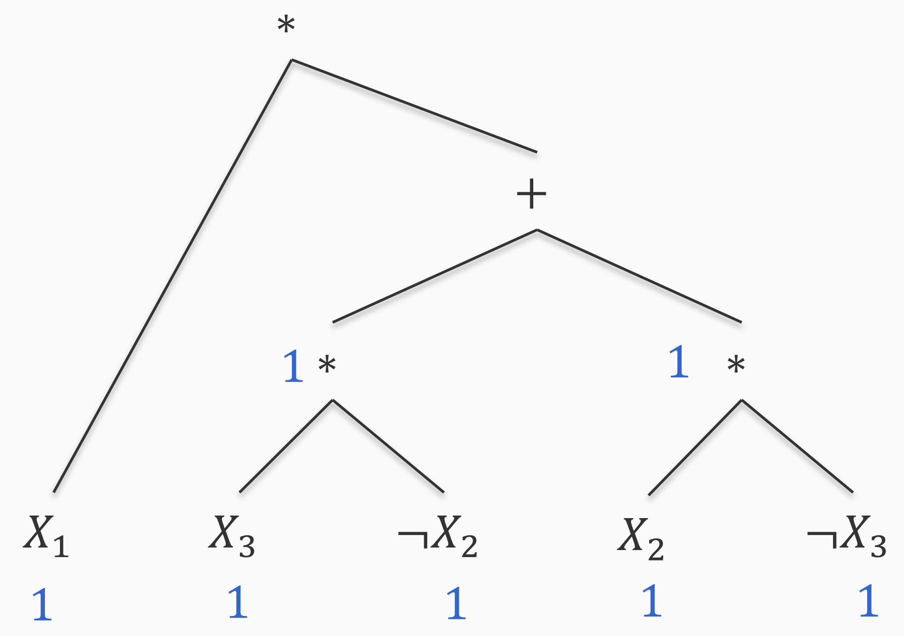

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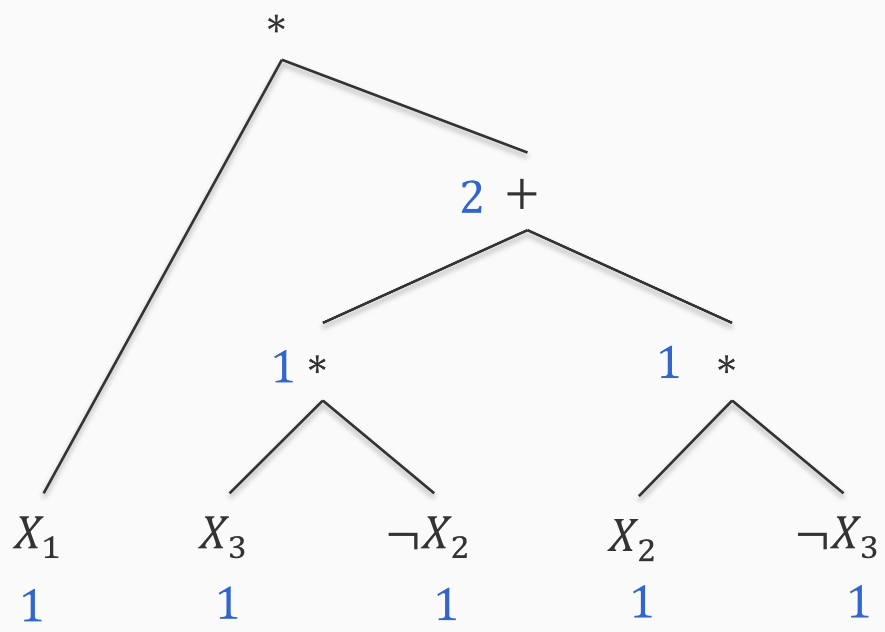

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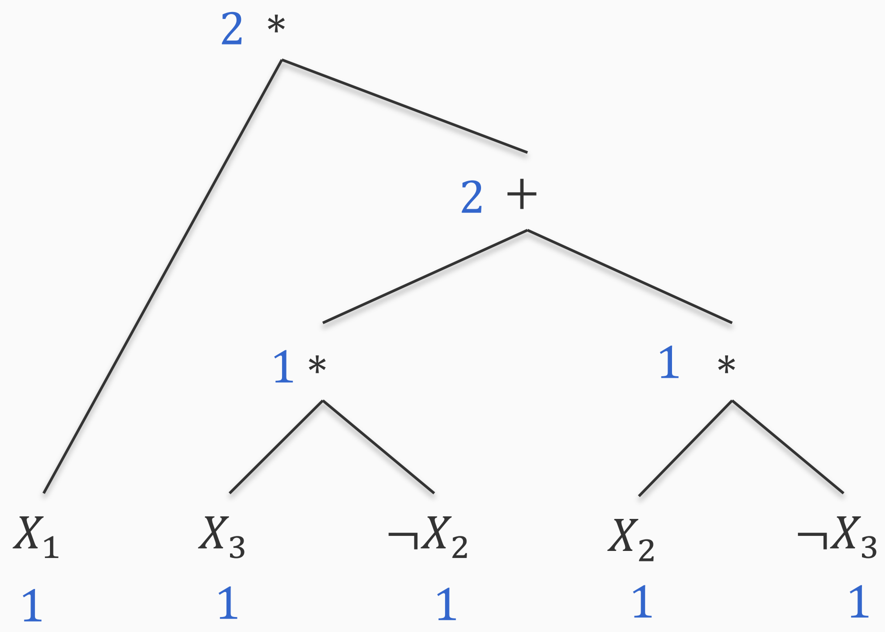

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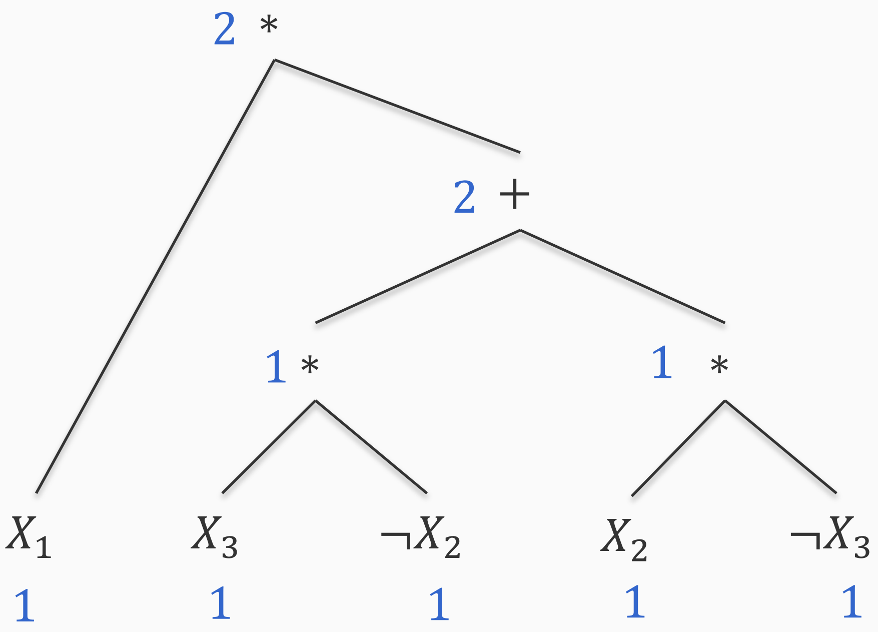

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The value at the root is the number of satisfying assignments of the formula!

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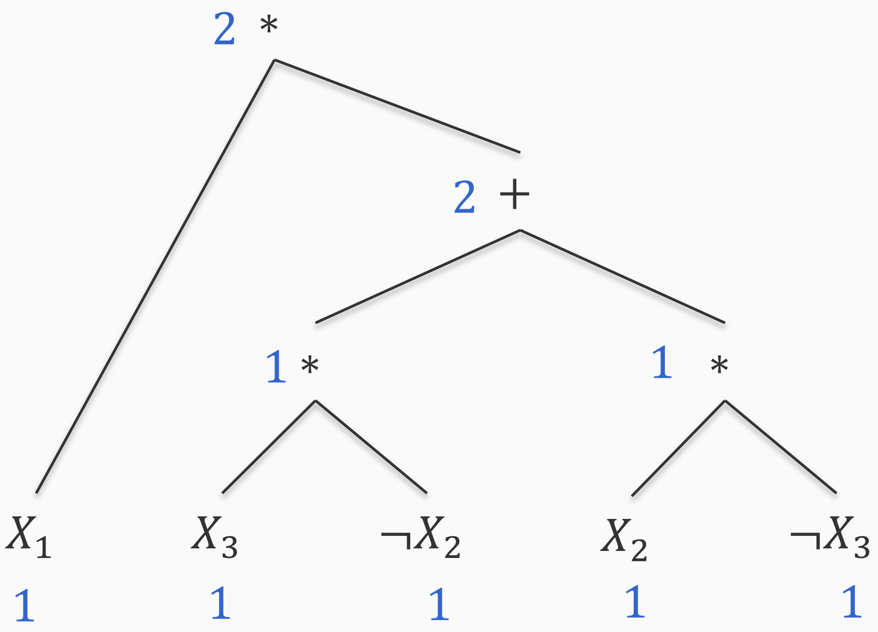

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Key point: We can count the number of satisfying assignments with what looks like a forward pass of a neural network

Efficient, if:
1. The network (i.e. the d-DNNF) can be constructed efficiently
2. The network is not too large

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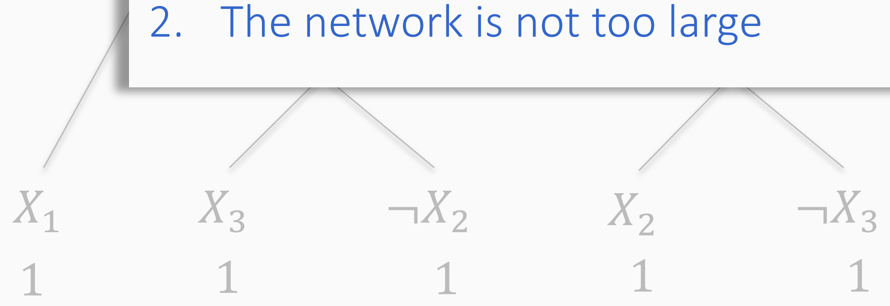

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We can also perform weighted model counting

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Suppose we have weights for variables

$\boldsymbol{X}$	$\mathbf{w}\text{eight}(\boldsymbol{X})$	$\mathbf{w}\text{eight}(\neg\boldsymbol{X})$
X_1	0.8	0.2
X_2	0.3	0.7
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Let’s look at the truth table first

X_1	X_2	X_3	α
$\top$	$\top$	$\top$	$\bot$
$\top$	$\top$	$\bot$	$\top$
$\top$	$\bot$	$\top$	$\top$
$\top$	$\bot$	$\bot$	$\bot$
$\bot$	$\top$	$\top$	$\bot$
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X_1	X_2	X_3	α	weight
$\top$	$\top$	$\top$	$\bot$	
$\top$	$\top$	$\bot$	$\top$	$0.8 \times 0.3 \times 0.4 = 0.096$
$\top$	$\bot$	$\top$	$\top$	$0.8 \times 0.7 \times 0.6 = 0.336$
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Total weight
$= 0.096 + 0.336$
$= 0.432$

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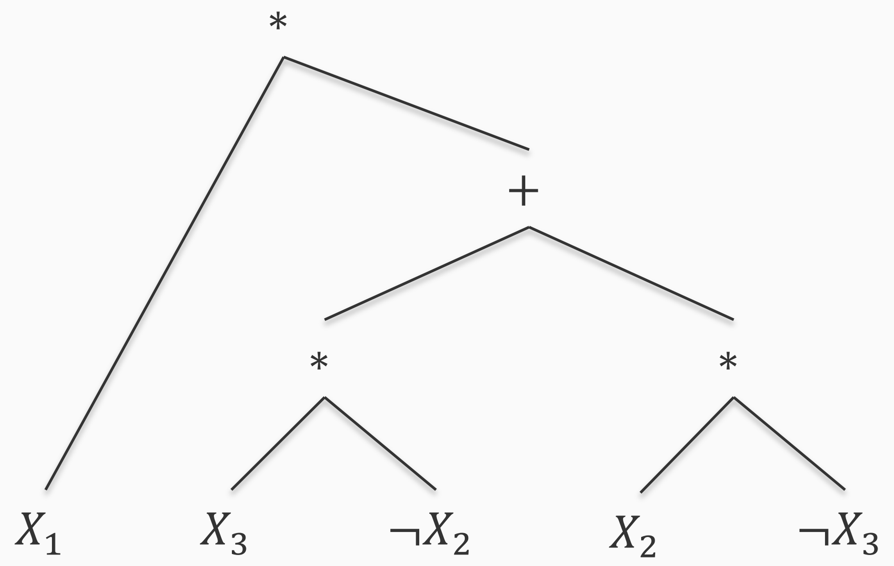
Total weight of satisfying assignments $= 0.432$

We can compute this weight using the counting tree



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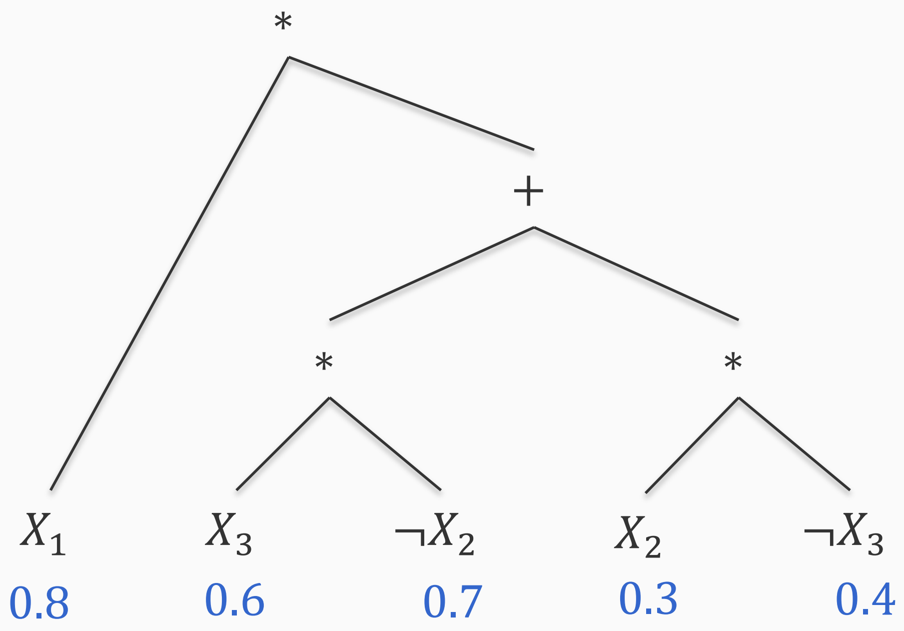

First, assign all the literals their respective weights

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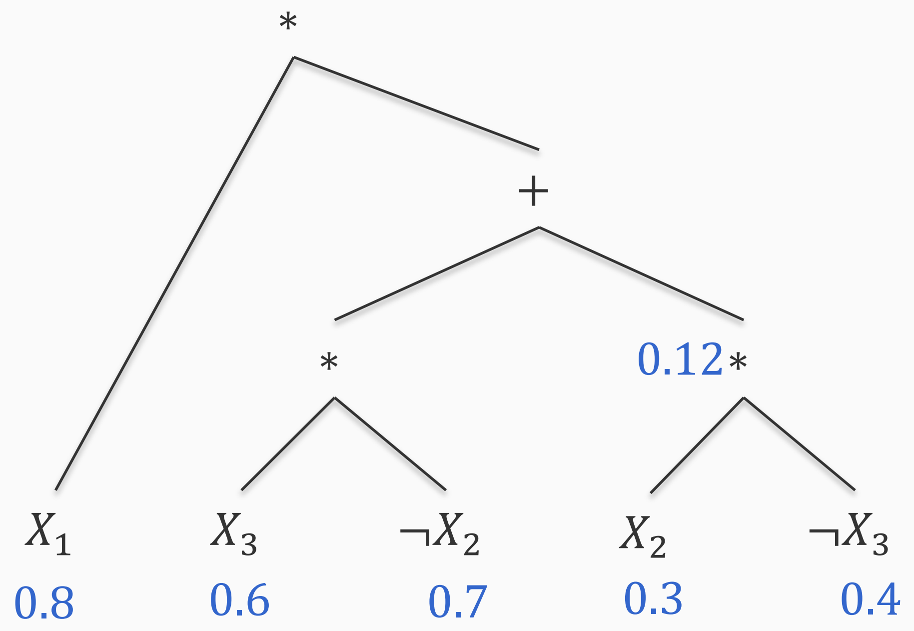

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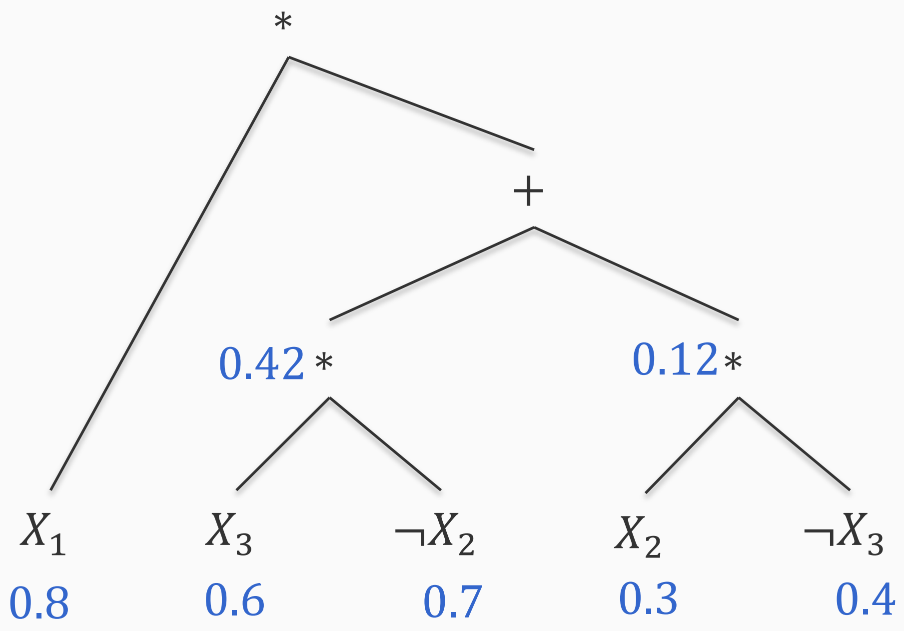

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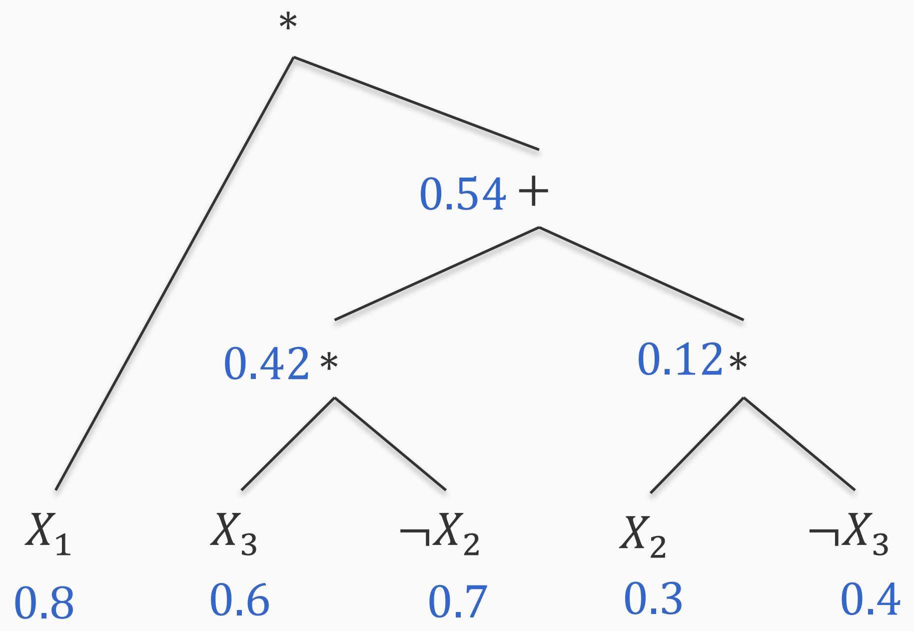

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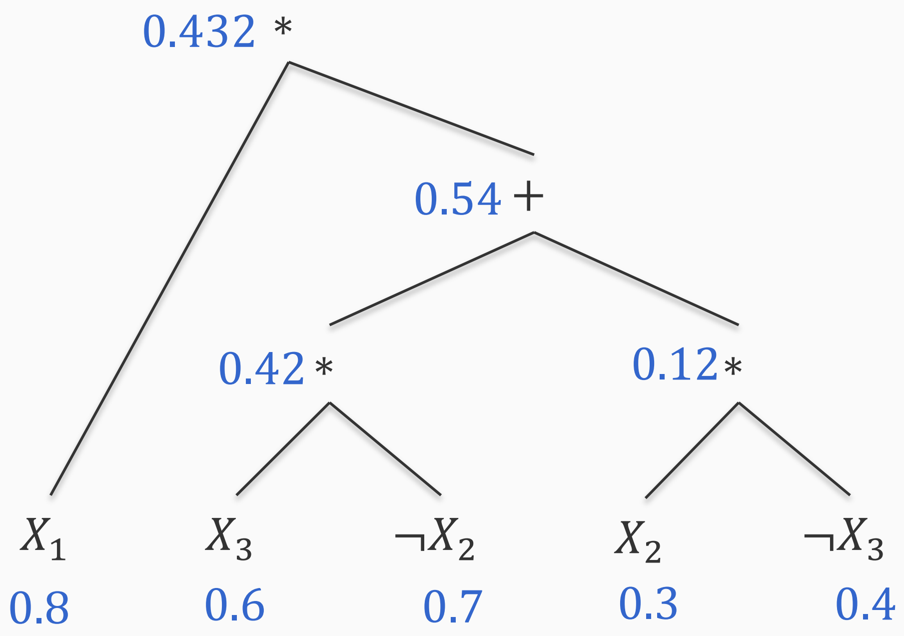

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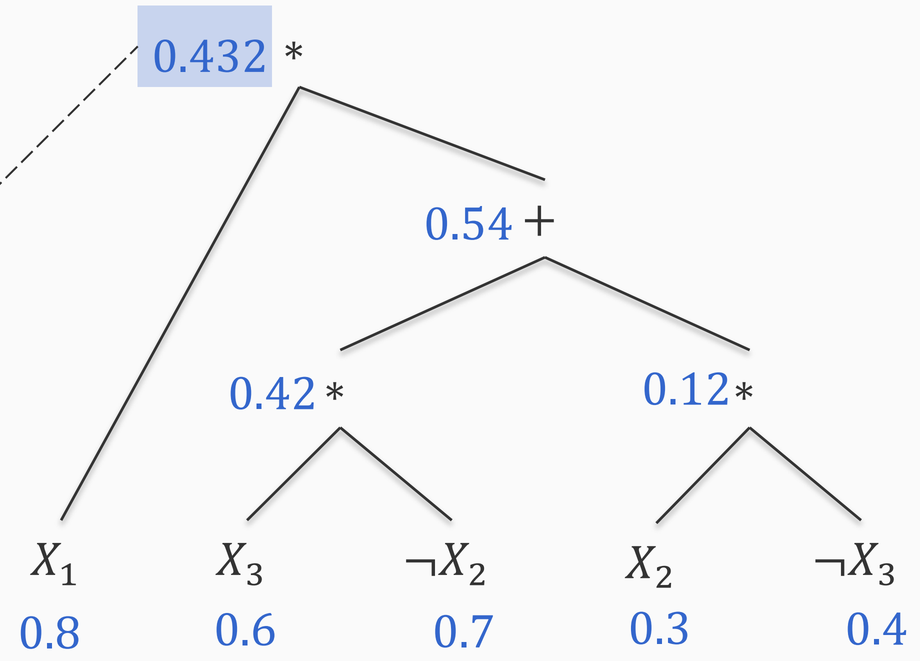

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Next, propagate forward

The value at the root is the total weighted model count!

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A second look at the semantic loss

$$L(\alpha, \mathbf{p}) \propto -\log \sum_{x \models \alpha} \left(\prod_{i : x \models X_i} p_i \cdot \prod_{i : x \models \neg X_i} (1 - p_i) \right)$$

This term requires us to accumulate quantities computed for every satisfying assignment for the formula α

Is this a problem? Have we seen this before?

Computing the semantic loss requires us to perform weighted model counting

Intractable in the worst case, *but tractable subsets of logic exist*

A second look at the semantic loss

$$L(\alpha, \mathbf{p}) \propto -\log \boxed{\sum_{x \models \alpha} \left(\prod_{i : x \models X_i} p_i \cdot \prod_{i : x \models \neg X_i} (1 - p_i) \right)}$$

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$$L(\alpha, \mathbf{p}) \propto -\log \boxed{WMC(\alpha, \mathbf{p})}$$

Knowledge compilation can help

Knowledge compilation: The process of converting a propositional knowledge base into a form that better supports certain kinds of queries

In our case, we can compile any propositional formula into a d-DNNF, which allows for two operations efficiently (if the resulting d-DNNF is not too large):

1. We can perform model counting and weighted model counting efficiently

2. We can take partial derivatives with respect to inputs efficiently

Logic as loss: Semantic loss

- Building up to semantic loss: The axioms
- Semantic loss
- Examples
  - Conjunction
  - Implication
- Complex constraints & Weighted Model Counting
  - Knowledge Compilation
  - Example: The exactly-one constraint

A simple semi-supervised learning example

Suppose we have:

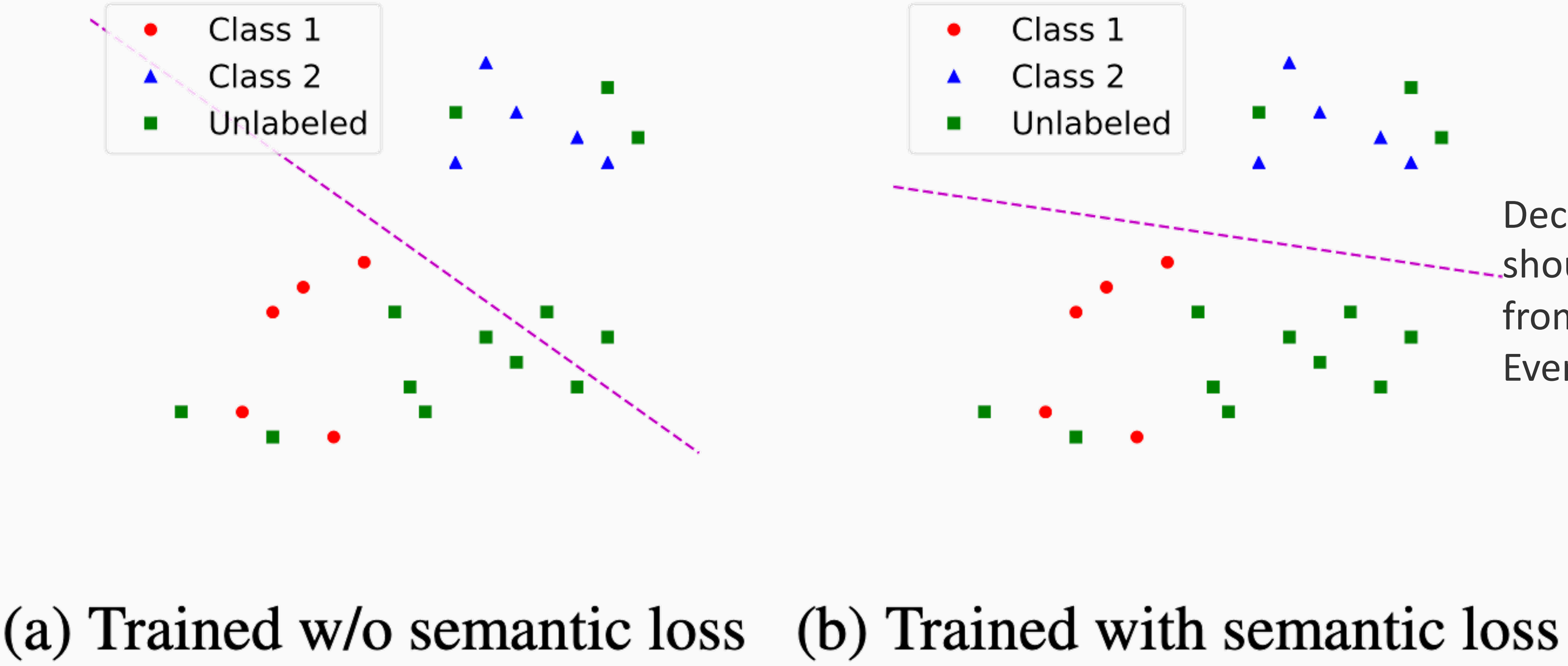
- A small number of labeled examples for a task with k labels
- A large collection of unlabeled examples

What information can the unlabeled examples provide to a model?

An unlabeled example must also have one and exactly one of the k labels

Can this information help train a model?

A binary classification example



Decision boundaries should be far away from all examples. Even unlabeled ones

Xu, Jingyi, Zilu Zhang, Tal Friedman, Yitao Liang, and Guy Van den Broeck. "A semantic loss function for deep learning with symbolic knowledge." In *ICML 2018*

The exactly-one constraint

Suppose we have three possible decisions produced by one or more neural networks: X_1, X_2, X_3

We want to enforce the following constraints about these decisions:

- One of these three decisions must be `true`

$$X_1 \vee X_2 \vee X_3$$

- No two of the three decisions can simultaneously be `true`

$$\neg X_1 \vee \neg X_2$$
$$\neg X_2 \vee \neg X_3$$
$$\neg X_3 \vee \neg X_1$$

Together, these constraints require that exactly one of the decisions should be `true`

How can we incorporate this knowledge into our loss?

The compiled exactly-one constraint

The original constraint

$$(X_1 \lor X_2 \lor X_3) \land (\neg X_1 \lor \neg X_2) \land (\neg X_2 \lor \neg X_3) \land (\neg X_3 \lor \neg X_1)$$

This is not in the deterministic decomposable negation normal form

The compiled exactly-one constraint

The original constraint

$$(X_1 \vee X_2 \vee X_3) \wedge (\neg X_1 \vee \neg X_2) \wedge (\neg X_2 \vee \neg X_3) \wedge (\neg X_3 \vee \neg X_1)$$

This is not in the deterministic decomposable negation normal form

But we can compile it to produce the following equivalent d-DNNF expression

$$(X_1 \wedge \neg X_2 \wedge \neg X_3) \vee (\neg X_1 \wedge X_2 \wedge \neg X_3) \vee (\neg X_1 \wedge \neg X_2 \wedge X_3)$$

Refer: The work by Adnan Darwiche in the 2000s that defines the normal form, analyzes complexity of querying it, and shows how to convert arbitrary Boolean formulas to the form

The compiled exactly-one constraint

The original constraint

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But we can compile it to produce the following equivalent d-DNNF expression

$$(X_1 \wedge \neg X_2 \wedge \neg X_3) \vee (\neg X_1 \wedge X_2 \wedge \neg X_3) \vee (\neg X_1 \wedge \neg X_2 \wedge X_3)$$

Refer: The work by Adnan Darwiche in the 2000s that defines the normal form, analyzes complexity of querying it, and shows how to convert arbitrary Boolean formulas to the form

Important: Because the semantic loss does not depend on the syntactic form that we use to define the constraint, we are free to use the more efficient form

Summary: Semantic loss

An axiomatic approach for converting logic to loss functions

- Produces differentiable losses
- Equivalent to cross-entropy when we have labeled examples

Key technical component

- Sum over the probabilities of assignments that satisfy the Boolean expression
- In practice: compile to tractable representations, and if this produces a small enough expression, we can perform forward and backward passes using standard tools
- Other approaches possible. E.g. approximation

Pros and cons

- Well defined semantics, syntactic variations don't matter
- But, could hide a difficult computational problem in the innermost loop of gradient based optimization