Symbolic Logic: An Introduction

THE UNIVERSITY OF UTAH

Inference & Inference rules

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If the knowledge base contains something that matches α,

then the sentence β is logically entailed

Inference rule: Modus Ponens

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From an implication, and the premise of the implication, we can infer the conclusion

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2. It is raining

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Modus Ponens allows us to conclude that the ground is wet

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$$\frac{\alpha \land \beta, \qquad \neg\beta}{\alpha}$$

Unit Resolution: If one of the disjuncts β of a disjunction is false, then for the disjunction to hold, the other disjunct α should be true

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$$\frac{\alpha_1 \wedge \alpha_2 \wedge \cdots \wedge \alpha_n}{\alpha_i}$$

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Resolution: The formula β can't be both true and false. So if both $\alpha \wedge \beta$ and $\neg\beta \wedge \gamma$ hold, then one of α or γ should hold

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And there are more inference rules