Symbolic Logic: An Introduction

THE UNIVERSITY OF UTAH

Propositional Logic: Syntax

Propositional logic: Syntax

Propositional logic consists of sentences that have a truth value (either **`true`** or **`false`**)

There are three kinds of sentences:

1. atoms,
2. literals, and,
3. formulas

Sentences in propositional logic:
1. Atoms

Any symbol that cannot be decomposed further is called an atom

Sentences in propositional logic:
1. Atoms

Any symbol that cannot be decomposed further is called an atom

There are two types of atoms:

Sentences in propositional logic:
1. Atoms

Any symbol that cannot be decomposed further is called an atom

There are two types of atoms:

1. Constants: The symbols $\top$ (denoting **`true`**) and $\bot$ (denoting **`false`**)

Sentences in propositional logic: 1. Atoms

Any symbol that cannot be decomposed further is called an atom

There are two types of atoms:

1. Constants: The symbols ⊤ (denoting **`true`**) and ⊥ (denoting **`false`**)

2. Propositional variables: symbols whose truth values may not be known
   - Typically denoted by letters (`p`, `q`, `r`, ...) or more descriptive names (`IsItWinter`, `IsFivePrime`, ...)
   - These will help build the complex propositional sentences

Sentences in propositional logic:
2. Literals

A literal is either an atom or its negation

If α is any atom, then both α and $\neg\alpha$ are called literals

Examples of literals: $p, \neg p, q, \neg q, \ldots$

Not literals: $\mathrm{p} \land q, \neg q \lor r, \ldots$

Sentences in propositional logic:
3. Propositional Formulas

Formulas use various logical connectives to build up complex sentences

Sentences in propositional logic:
3. Propositional Formulas

Formulas use various logical connectives to build up complex sentences

- Every literal is a formula

Sentences in propositional logic: 3. Propositional Formulas

Formulas use various logical connectives to build up complex sentences

- Every literal is a formula
- If F is a formula, then its negation $\neg F$ is a formula, read as “*not F*”

Sentences in propositional logic:
3. Propositional Formulas

Formulas use various logical connectives to build up complex sentences

- Every literal is a formula
- If F is a formula, then its negation $\neg F$ is a formula, read as "*not F*"
- If A and B are formulas, then their conjunction, denoted by $A \wedge B$ is a formula, read as "*A and B*"

Sentences in propositional logic: 3. Propositional Formulas

Formulas use various logical connectives to build up complex sentences

- Every literal is a formula
- If F is a formula, then its negation $\neg F$ is a formula, read as “*not F*”
- If A and B are formulas, then their conjunction, denoted by $A \wedge B$ is a formula, read as “*A and B*”
- If A and B are formulas, then their disjunction, denoted by $A \vee B$ is a formula, read as “*A or B*”

Sentences in propositional logic: 3. Propositional Formulas

Formulas use various logical connectives to build up complex sentences

- Every literal is a formula
- If F is a formula, then its negation $\neg F$ is a formula, read as "*not F*"
- If A and B are formulas, then their conjunction, denoted by $A \wedge B$ is a formula, read as "*A and B*"
- If A and B are formulas, then their disjunction, denoted by $A \vee B$ is a formula, read as "*A or B*"
- If A and B are formulas, then the implication, denoted by $A \rightarrow B$ is a formula, read as "*A implies B*"

Sentences in propositional logic:
3. Propositional Formulas

Formulas use various logical connectives to build up complex sentences

- Every literal is a formula
- If F is a formula, then its negation $\neg F$ is a formula, read as "*not F*"
- If A and B are formulas, then their conjunction, denoted by $A \wedge B$ is a formula, read as "*A and B*"
- If A and B are formulas, then their disjunction, denoted by $A \vee B$ is a formula, read as "*A or B*"
- If A and B are formulas, then the implication, denoted by $A \rightarrow B$ is a formula, read as "*A implies B*"
- If A and B are formulas, then the double implication, denoted by $A \leftrightarrow B$ is a formula, read as "*A if, and only, B*"

Well-formed formulas

Using atomic symbols, well-formed formulas can be created by repeated and appropriate application of the operators $\neg$, $\lor$, $\land$, $\rightarrow$, $\leftrightarrow$

Well-formed formulas

Using atomic symbols, well-formed formulas can be created by repeated and appropriate application of the operators $\neg,\ \vee,\ \wedge,\ \rightarrow,\ \leftrightarrow$

Examples of well-formed formulas

$\top$

$\bot$

P_1

$\neg P_1$

$P_1 \rightarrow P_2$

$P_1 \rightarrow (\neg P_2 \vee P_3)$

$P_1 \vee (P_2 \wedge \neg P_3)$

$\Big(\big((P_1 \rightarrow P_2) \rightarrow P_2\big) \rightarrow \neg P_1\Big) \rightarrow \neg P_2$

Well-formed formulas

Using atomic symbols, well-formed formulas can be created by repeated and appropriate application of the operators $\neg$, $\vee$, $\wedge$, $\rightarrow$, $\leftrightarrow$

Examples of well-formed formulas

$\top$

$\bot$

P_1

$\neg P_1$

$P_1 \rightarrow P_2$

$P_1 \rightarrow (\neg P_2 \vee P_3)$

$P_1 \vee (P_2 \wedge \neg P_3)$

$\left(\left((P_1 \rightarrow P_2) \rightarrow P_2\right) \rightarrow \neg P_1\right) \rightarrow \neg P_2$

Examples of ill-formed formulas

$\neg \vee P_1$

$P_2 \vee\wedge P_1$

$\neg\neg \wedge P_1$