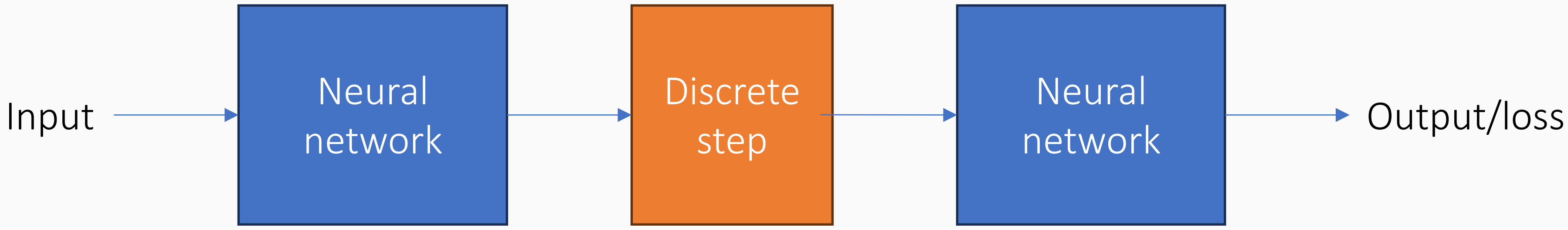
Learning with symbols within neural networks: Straight-through estimators

Neuro-symbolic modeling

THE UNIVERSITY OF UTAH

Neural networks containing discrete elements



Let's see some examples

This lecture

- Motivating examples
- The straight-through estimator
- The Gumbel trick
- REINFORCE

(others if time permits)

Not all these approaches are always applicable

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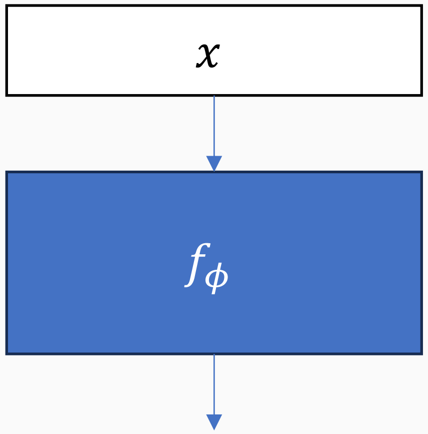

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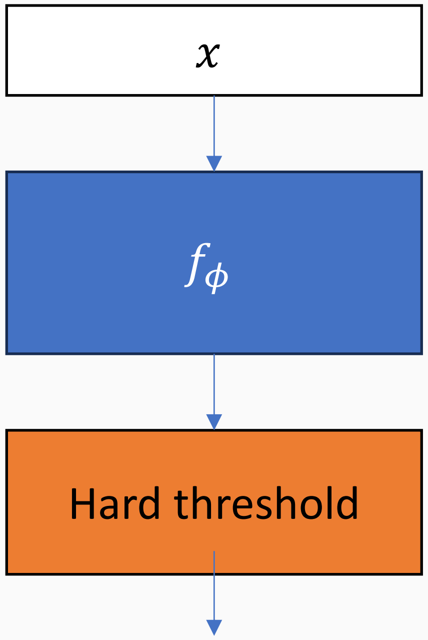
Each score is thresholded at zero to produce a d-dimensional binary vector z



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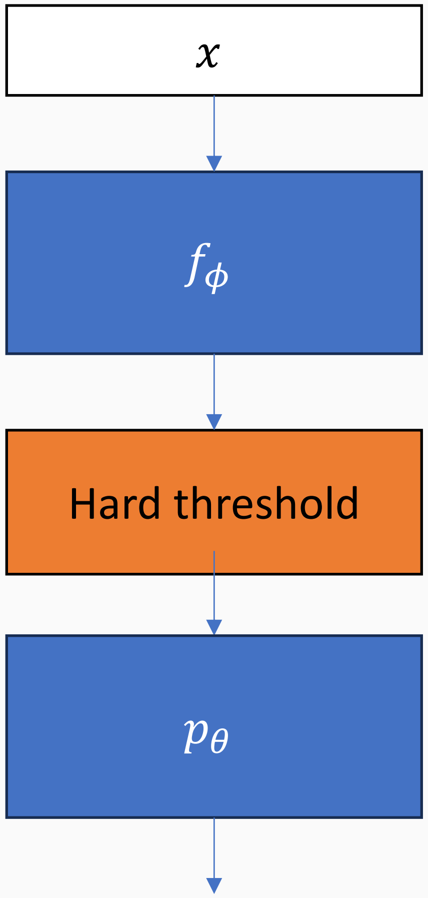
The final output is then $g_\theta(z)$



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Let us write the derivative of this loss with respect to the parameters:

$$\nabla_\theta \mathrm{L} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial \theta}$$

This part is standard: L and y are differentiable functions of the parameters θ

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This is not useful because $\frac{\partial z}{\partial f}$ is zero almost everywhere and infinite at $f = 0$

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For the backward pass alone, pretend that z is the result identity function

Hinton, Geoffrey. “Neural networks for machine learning”. *Coursera, lecture 15b* (2012).
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In the forward pass, it still uses the threshold function

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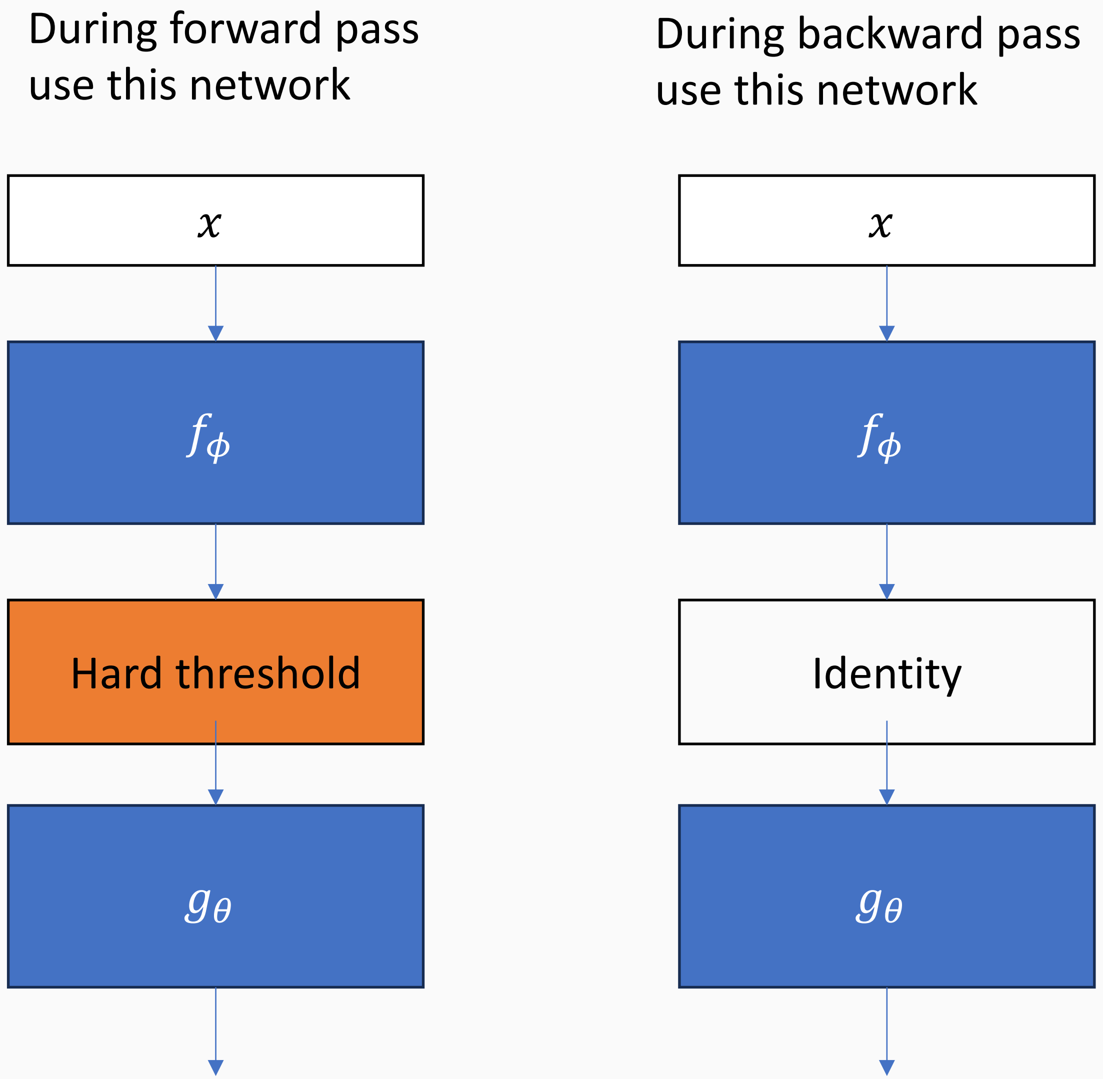

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This gradient estimator is poorly motivated

Yet, it sometimes works! (And easy to implement)

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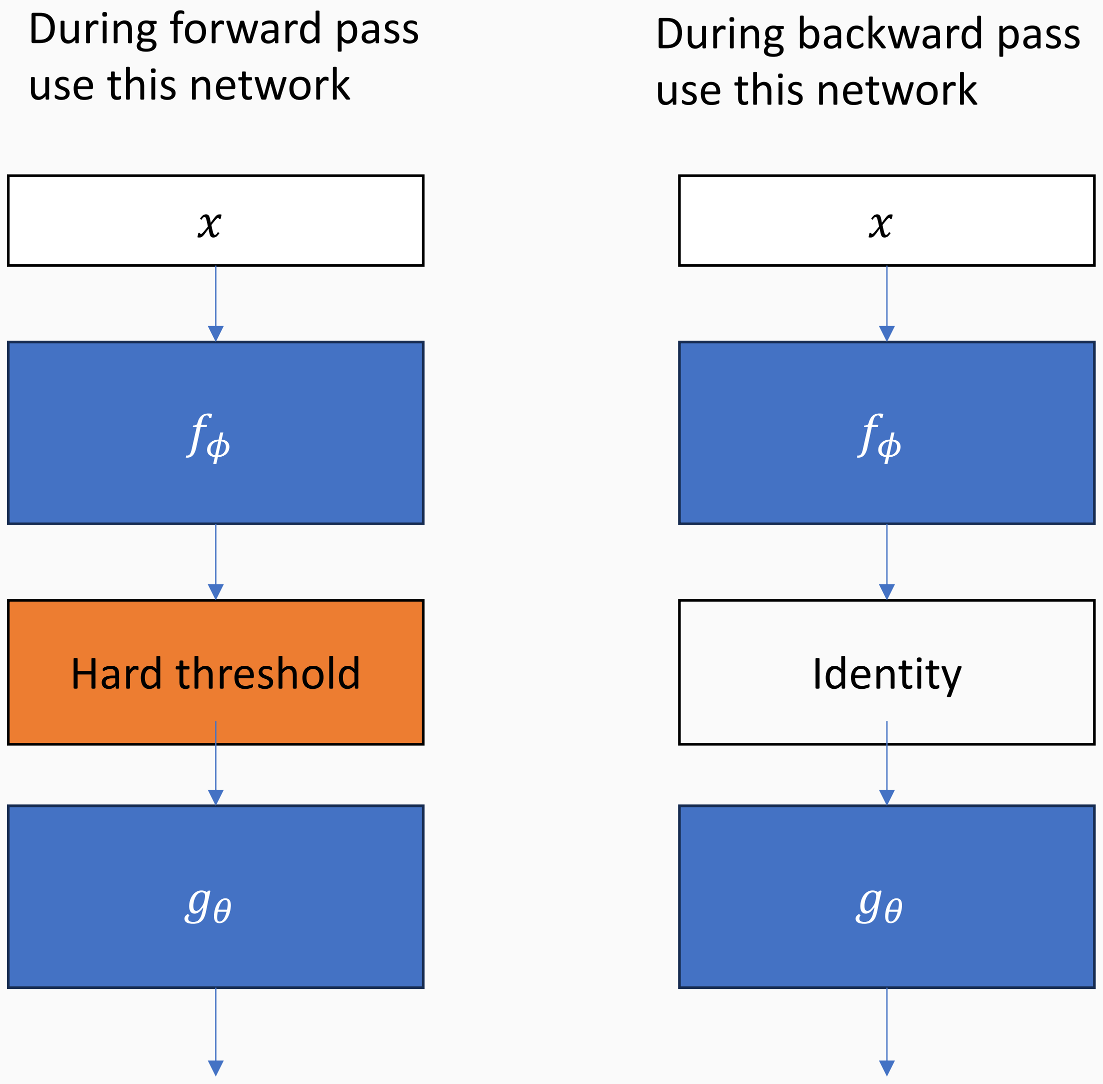

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One failure case: When there are dependencies between the binary variables, these are not accounted for in the gradient