Learning with weak supervision
What we have seen so far...

- What is structured output?
- Joint scoring functions over multiple interacting decisions
- Various families of inference algorithms
  - Search over a combinatorial space
- Learning in the fully supervised setting
The difficulty with supervised learning

Annotated data is expensive and costs increase when...

- ...a task requires specialized expertise
  E.g. “Only a trained linguist or a board certified radiologist can label my data”

- ...labeling examples involves making multiple decisions
  E.g. “Annotate this sentence with a parse tree”
  (instead of a single binary decision)
The difficulty with supervised learning

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- ...*a task requires specialized expertise*
  
  E.g. “Only a trained linguist or a board certified radiologist can label my data”

- ...*labeling examples involves making multiple decisions*
  
  E.g. “Annotate this sentence with a parse tree”
  (instead of a single binary decision)

Creating labeled examples for structured output problems is expensive and time consuming
What if: The labels are missing

Training data: \[ D = \{(x_i, y_i)\} \]

Or perhaps we have

A small number of labeled examples

+ Extra information in the form of unlabeled examples, or constraints
This lecture

• What is weak supervision?

• The expectation maximization algorithm
  – A general template for learning with weak supervision

• Learning with latent variables

• Learning with constraints
This lecture

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Weak supervision: The motivation

What can we do with unlabeled or partially labeled examples?

Assume that we know what the task is, that is, the definition of the structure in question

Example:

• Suppose we know that my task involves predicting a sequence of labels, but...
• ...we don’t have any labeled data
The many forms of supervision

- **Labeled examples**
  - We have examples with the output structure labeled

- **Unlabeled/partially labeled examples**
  - We have examples, and perhaps also parts of the output structures for some of them

- **Hard constraints**
  - Restrict the space of output structures that can exist

- **Soft constraints**
  - Similar to hard constraints, but allows violations

- **Distant/indirect supervision**
  - We know of another task that is correlated in a well defined way with the task we care about

- **Heuristics**
  - We can write simple programs that are reasonably good on specific examples
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Other kinds of supervision exist.
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Other kinds of supervision exist.

Usually we have a mix of different kinds of supervision.
The many forms of supervision

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Other kinds of supervision exist.

Usually we have a mix of different kinds of supervision.

How do we systematically take advantage of such signals?
This lecture

• What is weak supervision?

• The expectation maximization algorithm
  – A general template for learning with weak supervision

• Learning with latent variables

• Learning with constraints
Expectation Maximization

- A meta-algorithm to estimate a probability distribution when some part of the output is missing
  - The entire output could be missing (i.e. unlabeled examples)
  - A part of the output could be missing (i.e., partially labeled examples)
Expectation Maximization

- A meta-algorithm to estimate a probability distribution when some part of the output is missing
  - The entire output could be missing (i.e. unlabeled examples)
  - A part of the output could be missing (i.e., partially labeled examples)

- Needs assumptions about the underlying probability distribution
  - Performance sensitive to the validity of this assumption (and also the initial guess of the parameters)

- Converges to a local maximum of the likelihood function
Let’s revisit maximum likelihood estimation

Given unlabeled examples $D = \{x_i\}$, we want to learn the parameters $\theta$ that defines a probability distribution $P_\theta(x_i, y_i)$.
Let’s revisit maximum likelihood estimation

Given unlabeled examples $D = \{x_i\}$, we want to learn the parameters $\theta$ that defines a probability distribution $P_\theta(x_i, y_i)$

Find parameters that maximize the likelihood (or equivalently log-likelihood) of the data

$$\text{log likelihood}(D \mid \theta) = \sum_i \log P(x_i \mid \theta)$$
Let’s revisit maximum likelihood estimation

Given unlabeled examples $D = \{x_i\}$, we want to learn the parameters $\theta$ that defines a probability distribution $P_{\theta}(x_i, y_i)$

Find parameters that maximize the likelihood (or equivalently log-likelihood) of the data

$$\log \text{likelihood}(D \mid \theta) = \sum_i \log P(x_i \mid \theta)$$

But our model doesn’t directly tell us about this probability. It only knows $P_{\theta}(x_i, y_i)$. 
Let’s revisit maximum likelihood estimation

Given unlabeled examples \( D = \{x_i\} \), we want to learn the parameters \( \theta \) that defines a probability distribution \( P_\theta(x_i, y_i) \)

Find parameters that maximize the likelihood (or equivalently log-likelihood) of the data

\[
\text{log likelihood}(D \mid \theta) = \sum_i \log P(x_i \mid \theta)
\]

*How do we state the probability term in terms of \( P_\theta(x_i, y_i) \)?*
Let’s revisit maximum likelihood estimation

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$$\log \text{likelihood}(D \mid \theta) = \sum_i \log P(x_i \mid \theta)$$

How do we state the probability term in terms of $P_\theta(x_i, y_i)$?

**Answer**: Marginalize the missing terms
Let’s revisit maximum likelihood estimation

Given unlabeled examples $D = \{x_i\}$, we want to learn the parameters $\theta$ that defines a probability distribution $P_{\theta}(x_i, y_i)$

Find parameters that maximize the likelihood (or equivalently log-likelihood) of the data

$$\log \text{likelihood}(D \mid \theta) = \sum_i \log P(x_i \mid \theta)$$

*How do we state the probability term in terms of $P_{\theta}(x_i, y_i)$?*

**Answer:** Marginalize the missing terms

$$\log \text{likelihood}(D \mid \theta) = \sum_i \log \sum_{y \in y(x_i)} P_{\theta}(x_i, y)$$
Let’s revisit maximum likelihood estimation

Given unlabeled examples $D = \{x_i\}$, we want to learn the parameters $\theta$ that defines a probability distribution $P_\theta(x_i, y_i)$

Find parameters that maximize the likelihood (or equivalently log-likelihood) of the data

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**How do we state the probability term in terms of $P_\theta(x_i, y_i)$?**

**Answer:** Marginalize the missing terms

$$\text{log likelihood}(D \mid \theta) = \sum_i \log \sum_{y \in y(x_i)} P_\theta(x_i, y)$$
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Given unlabeled examples $D = \{x_i\}$, we want to learn the parameters $\theta$ that defines a probability distribution $P_\theta(x_i, y_i)$

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**How do we state the probability term in terms of $P_\theta(x_i, y_i)$?**

**Answer:** Marginalize the missing terms

$$\text{log likelihood}(D \mid \theta) = \sum_i \log \sum_{y \in Y(x_i)} P_\theta(x_i, y)$$

Sum over all structures for the example $x_i$
Let’s revisit maximum likelihood estimation

Given unlabeled examples $D = \{x_i\}$, we want to learn the parameters $\theta$ that defines a probability distribution $P_\theta(x_i, y_i)$

Find parameters that maximize the likelihood (or equivalently log-likelihood) of the data

$$\log \text{likelihood}(D \mid \theta) = \sum_i \log \sum_{y \in y(x_i)} P_\theta(x_i, y)$$

Goal: Maximize this expression in terms of the parameters

This maximization is not easy. Sum inside log
Let us build an approximation

**What we want:** Maximize \( \text{log likelihood}(D \mid \theta) = \text{LL}(D \mid \theta) \)

\[
\text{LL}(D \mid \theta) = \sum_i \log \sum_{y \in y(x_i)} P_\theta(x_i, y)
\]

Why do we want to maximize this? Because this gives us the maximum likelihood estimate
Let us build an approximation

What we want: Maximize log likelihood $D \mid \theta = LL(D \mid \theta)$

$$LL(D \mid \theta) = \sum_i \log \sum_{y \in \mathcal{Y}(x_i)} P_\theta(x_i, y)$$

$$= \sum_i \log \sum_{y \in \mathcal{Y}(x_i)} \left( Q_i(y) \cdot \frac{P_\theta(x_i, y)}{Q_i(y)} \right)$$

This is true for any probability distribution $Q_i(y)$.
Let us build an approximation

What we want: Maximize log likelihood \( D \mid \theta = \text{LL}(D \mid \theta) \)

\[
\text{LL}(D \mid \theta) = \sum_{i} \log \sum_{y \in \mathcal{y}(x_i)} P_{\theta}(x_i, y) = \sum_{i} \log \sum_{y \in \mathcal{y}(x_i)} \left( Q_i(y) \cdot \frac{P_{\theta}(x_i, y)}{Q_i(y)} \right)
\]

This is true for any probability distribution \( Q_i(y) \).

The summation over \( y \) is the definition of expectation with respect to \( Q_i(y) \)

\[
E_{z \sim Q}[f(z)] = \sum_{z} Q(z)f(z)
\]
Let us build an approximation

**What we want:** Maximize log likelihood \( D \mid \theta \) = LL(D \mid \theta)

\[
LL(D \mid \theta) = \sum_i \log \left( \sum_{y \in y(x_i)} P_{\theta}(x_i, y) \right)
\]

\[
= \sum_i \log \left( \sum_{y \in y(x_i)} \left( Q_i(y) \cdot \frac{P_{\theta}(x_i, y)}{Q_i(y)} \right) \right)
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\[
E_{z \sim Q}[f(z)] = \sum_z Q(z)f(z)
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Let us build an approximation

**What we want:** Maximize log likelihood \( D \mid \theta \) = LL(\( D \mid \theta \))

\[
\begin{align*}
\text{LL}(D \mid \theta) &= \sum_i \log \sum_{y \in y(x_i)} P_\theta(x_i, y) \\
&= \sum_i \log \sum_{y \in y(x_i)} \left( Q_i(y) \cdot \frac{P_\theta(x_i, y)}{Q_i(y)} \right) \\
E_{z \sim Q}[f(z)] &= \sum_z Q(z)f(z)
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\text{LL}(D \mid \theta) &= \sum_i \log \sum_{y \in y(x_i)} P_{\theta}(x_i, y) \\
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E_{z \sim Q}[f(z)] = \sum_z Q(z)f(z)
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Let us build an approximation

What we want: Maximize log likelihood \( D \mid \theta = LL(D \mid \theta) \)

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LL(D \mid \theta) = \sum_i \log \sum_{y \in y(x_i)} P_\theta(x_i, y)
= \sum_i \log \sum_{y \in y(x_i)} \left( Q_i(y) \cdot \frac{P_\theta(x_i, y)}{Q_i(y)} \right)
= \sum_i \log E_{y \sim Q_i} \left[ \frac{P_\theta(x_i, y)}{Q_i(y)} \right]

How do we proceed now?
We haven’t made the problem any simpler by just rewriting it.
Jensen’s inequality

If $f$ is a **convex** function and $X$ is a random variable, then

$$f(E[X]) \leq E[f(X)]$$

Or: If $f$ is a **concave** function and $X$ is a random variable, then

$$f(E[X]) \geq E[f(X)]$$
Let’s apply Jensen’s inequality

If $f$ is a concave function and $X$ is a random variable, then

$$f(E[X]) \geq E[f(X)]$$
Let’s apply Jensen’s inequality

If $f$ is a concave function and $X$ is a random variable, then

$$f(E[X]) \geq E[f(X)]$$

Let us apply this to the following function:

$$\log E_{y \sim Q} \left[ \frac{P_{\theta}(x_i, y)}{Q_i(y)} \right]$$
Let’s apply Jensen’s inequality

If \( f \) is a concave function and \( X \) is a random variable, then
\[
f(E[X]) \geq E[f(X)]
\]

Let us apply this to the following function:
\[
\log E_{y \sim Q_i} \left[ \frac{P_\theta(x_i, y)}{Q_i(y)} \right]
\]

\( \log(x) \) is a concave function in \( x \) and \textit{the expression inside the expectation} is a random variable
\[
\log E_{y \sim Q_i} \left[ \frac{P_\theta(x_i, y)}{Q_i(y)} \right] \geq E_{y \sim Q_i} \left[ \log \frac{P_\theta(x_i, y)}{Q_i(y)} \right]
\]
Let us build an approximation

**What we want:** Maximize $\log \text{likelihood}(D \mid \theta) = \text{LL}(D \mid \theta)$

\[
\text{LL}(D \mid \theta) = \sum_i \log \sum_{y \in y(x_i)} P_\theta(x_i, y)
\]

\[
= \sum_i \log \sum_{y \in y(x_i)} \left( Q_i(y) \cdot \frac{P_\theta(x_i, y)}{Q_i(y)} \right)
\]

\[
= \sum_i \log \mathbb{E}_{y \sim Q_i} \left[ \frac{P_\theta(x_i, y)}{Q_i(y)} \right]
\]

By Jensen’s inequality:

\[
\log \mathbb{E}_{y \sim Q_i} \left[ \frac{P_\theta(x_i, y)}{Q_i(y)} \right] \geq \mathbb{E}_{y \sim Q_i} \left[ \log \frac{P_\theta(x_i, y)}{Q_i(y)} \right]
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Let us build an approximation

What we want: Maximize log likelihood \( D \mid \theta \) = \( LL(D \mid \theta) \)

\[
LL(D \mid \theta) = \sum_i \log \left( \sum_{y \in \mathcal{Y}(x_i)} P_\theta(x_i, y) \right)
\]

\[
= \sum_i \log \left( \sum_{y \in \mathcal{Y}(x_i)} Q_i(y) \cdot \frac{P_\theta(x_i, y)}{Q_i(y)} \right)
\]

\[
= \sum_i \log E_{y \sim Q_i} \left[ \frac{P_\theta(x_i, y)}{Q_i(y)} \right]
\]

\[
\geq \sum_i E_{y \sim Q_i} \left[ \log \frac{P_\theta(x_i, y)}{Q_i(y)} \right]
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\]
Let us build an approximation

**What we want:** Maximize log likelihood \( (D \mid \theta) = LL(D \mid \theta) \)

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LL(D \mid \theta) = \sum_i \log \sum_{y \in \mathcal{Y}(x_i)} P_\theta(x_i, y)
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\[
= \sum_i \log \sum_{y \in \mathcal{Y}(x_i)} \left( Q_i(y) \cdot \frac{P_\theta(x_i, y)}{Q_i(y)} \right)
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\]

\[
= \sum_i E_{y \sim Q_i} [\log P_\theta(x_i, y)] - \sum_i E_{y \sim Q_i} [\log Q_i(y)]
\]

Rewrite log + linearity of expectation
Let us build an approximation

**What we want**: Maximize log likelihood \( D \mid \theta \) = LL(D \mid \theta)

\[
\begin{align*}
\text{LL}(D \mid \theta) &= \sum_i \log \sum_{y \in y(x_i)} P_\theta(x_i, y) \\
&= \sum_i \log \sum_{y \in y(x_i)} \left( Q_i(y) \cdot \frac{P_\theta(x_i, y)}{Q_i(y)} \right) \\
&= \sum_i \log E_{y \sim Q_i} \left[ \frac{P_\theta(x_i, y)}{Q_i(y)} \right] \\
&\geq \sum_i E_{y \sim Q_i} \left[ \log \frac{P_\theta(x_i, y)}{Q_i(y)} \right] \\
&= \sum_i E_{y \sim Q_i} [\log P_\theta(x_i, y)] - \sum_i E_{y \sim Q_i} [\log Q_i(y)]
\end{align*}
\]
Let us build an approximation

What we want: Maximize log likelihood \( D \mid \theta \) = LL(D \mid \theta)

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\text{LL}(D \mid \theta) = \sum_i \log \sum_{y \in \mathcal{Y}(x_i)} P_\theta(x_i, y)
= \sum_i \log \sum_{y \in \mathcal{Y}(x_i)} \left( Q_i(y) \cdot \frac{P_\theta(x_i, y)}{Q_i(y)} \right)
\]

The strategy: Let us maximize this lower bound on the likelihood instead

\[
\geq \sum_i E_{y \sim Q_i} \left[ \log \frac{P_\theta(x_i, y)}{Q_i(y)} \right]
= \sum_i E_{y \sim Q_i} [\log P_\theta(x_i, y)] - \sum_i E_{y \sim Q_i} [\log Q_i(y)]
\]
Expectation Maximization: The strategy

What we want (but can’t have)

\[
\text{log likelihood}(D \mid \theta) = \sum_i \log \sum_{y \in y(x_i)} P_\theta(x_i, y)
\]

The strategy: Think of log probabilities as random variables

Learn by repeatedly maximizing a lower bound of the log likelihood

\[
F(\theta, Q_i) = \sum_i E_{y \sim Q_i} [\log P_\theta(x_i, y)] - \sum_i E_{y \sim Q_i} [\log Q_i(y)]
\]
Expectation Maximization: The strategy

Learning by maximizing expected log likelihood of the data

\[
F(\theta, Q_i) = \sum_i E_{y \sim Q_i} [\log P_\theta(x_i, y)] - \sum_i E_{y \sim Q_i} [\log Q_i(y)]
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Expectation Maximization: The strategy

Learning by maximizing expected log likelihood of the data

\[
F(\theta, Q_i) = \sum_i E_{y \sim Q_i} [\log P_\theta(x_i, y)] - \sum_i E_{y \sim Q_i} [\log Q_i(y)]
\]

Still need to decide what is a good \( Q_i \)

What we would like is the one that makes this lower bound tight

\[
\log E_{y \sim Q_i} \left[ \frac{P_\theta(x_i, y)}{Q_i(y)} \right] \geq E_{y \sim Q_i} \left[ \log \frac{P_\theta(x_i, y)}{Q_i(y)} \right]
\]

Jensen’s inequality
Expectation Maximization: The strategy

Learning by maximizing expected log likelihood of the data

\[ F(\theta, Q_i) = \sum_{i} E_{y \sim Q_i} [\log P_\theta(x_i, y)] - \sum_{i} E_{y \sim Q_i} [\log Q_i(y)] \]

Still need to decide what is a good \( Q_i \)
What we would like is the one that makes this lower bound tight

\[ \log E_{y \sim Q_i} \left[ \frac{P_\theta(x_i, y)}{Q_i(y)} \right] \geq E_{y \sim Q_i} \left[ \log \frac{P_\theta(x_i, y)}{Q_i(y)} \right] \]

Jensen’s inequality

We can show that if we had an estimate of the \( \theta \), say \( \theta^t \), then a tight lower bound is given by setting

\[ Q_i(y) = P_{\theta^t}(y \mid x_i) \]
Expectation Maximization

- Initialize the parameters $\theta^0$

- Return final $\theta$
Expectation Maximization

• Initialize the parameters $\theta^0$
• Repeat until convergence (t = 1, 2, ...)

• Return final $\theta$
Expectation Maximization

- Initialize the parameters $\theta^0$
- Repeat until convergence ($t = 1, 2, ...$)
  - **E-Step**: For every example $x_i$, estimate for every $y$
    \[ Q_i^t(y) \leftarrow P_{\theta^t}(y | x_i) \]
- Return final $\theta$
Expectation Maximization

- Initialize the parameters $\theta^0$
- Repeat until convergence ($t = 1, 2, ...$)
  - **E-Step**: For every example $x_i$, estimate for every $y$
    $$Q^t_i(y) \leftarrow P_{\theta^t}(y | x_i)$$
  
  - **M-Step**: Find $\theta^{t+1}$ by maximizing with respect to $\theta$
    $$F(\theta, Q^t) = \sum_i E_{y \sim Q^t_i} [\log P_{\theta}(x_i, y)] - \sum_i E_{y \sim Q^t_i} [\log Q^t_i(y)]$$

- Return final $\theta$
Expectation Maximization

- Initialize the parameters $\theta^0$
- Repeat until convergence ($t = 1, 2, ...$)
  
  - **E-Step**: For every example $x_i$, estimate for every $y$
    
    $$Q^t_i(y) \leftarrow P_{\theta^t}(y \mid x_i)$$

  - **M-Step**: Find $\theta^{t+1}$ by maximizing with respect to $\theta$
    
    $$F(\theta, Q^t) = \sum_i E_{y \sim Q^t_i}[\log P_{\theta}(x_i, y)] - \sum_i E_{y \sim Q^t_i}[\log Q^t_i(y)]$$

    Independent of $\theta$

- Return final $\theta$
Expectation Maximization

- Initialize the parameters $\theta^0$
- Repeat until convergence ($t = 1, 2, ...$
  - **E-Step**: For every example $x_i$, estimate for every $y$
    $$Q_i^t(y) \leftarrow P_{\theta^t}(y | x_i)$$
  - **M-Step**: Find $\theta^{t+1}$ by solving the maximization problem
    $$\theta^{t+1} \leftarrow \arg\max_{\theta} \sum_i E_{y \sim Q_i^t} [\log P_{\theta}(x_i, y)]$$
- Return final $\theta$
Expectation Maximization

• Initialize the parameters \( \theta^0 \)
• Repeat until convergence (\( t = 1, 2, ... \))
  – **E-Step**: For every example \( x_i \), estimate for every \( y \)
    \[
    Q^t_i(y) \leftarrow P_{\theta^t}(y \mid x_i)
    \]
  – **M-Step**: Find \( \theta^{t+1} \) by solving the maximization problem
    \[
    \theta^{t+1} \leftarrow \arg\max_{\theta} \sum_i E_{y \sim Q^t_i}[\log P_\theta(x_i, y)]
    \]
• Return final \( \theta \)

Intuitively: What is distribution over the latent variables for this set of parameters
Expectation Maximization

- Initialize the parameters $\theta^0$
- Repeat until convergence ($t = 1, 2, \ldots$)
  - **E-Step**: For every example $x_i$, estimate for every $y$
    \[ Q_i^t(y) \leftarrow P_{\theta^t}(y | x_i) \]
  - **M-Step**: Find $\theta^{t+1}$ by solving the maximization problem
    \[ \theta^{t+1} \leftarrow \arg\max_{\theta} \sum_i E_{y \sim Q_i^t} [\log P_{\theta}(x_i, y)] \]
- Return final $\theta$

Intuitively: What is distribution over the latent variables for this set of parameters

Intuitively: Using the current estimate for the hidden variables, what is the best set of parameters for the entire data
Expectation Maximization

- Initialize the parameters $\theta^0$
- Repeat until convergence ($t = 1, 2, ...$)
  - **E-Step**: For every example $x_i$, estimate for every $y$
    \[ Q_i^t(y) \leftarrow P_{\theta^t}(y \mid x_i) \]
    Given the parameters $\theta^t$, we can compute this function. Why?
  - **M-Step**: Find $\theta^{t+1}$ by solving the maximization problem
    \[ \theta^{t+1} \leftarrow \arg\max_{\theta} \sum_i E_{y \sim Q_i^t} [\log P_{\theta}(x_i, y)] \]
    This step needs can be solved either analytically or algorithmically.
- Return final $\theta$
Expectation Maximization

- Initialize the parameters $\theta^0$
- Repeat until convergence ($t = 1, 2, \ldots$)
  - E-Step: For every example $x_i$, estimate for every $y$
    $$Q^t_i(y) \leftarrow P_{\theta^t}(y \mid x_i)$$
  - M-Step: Find $\theta^t$ by solving the maximization problem
    $$\theta^t \leftarrow \arg\max \sum_{y} Q^t_i(y) \log P_{\theta^t}(x, y)$$
- Return final $\theta$

Given the parameters $\theta^t$, we can compute this function. Why?

EM is a meta-algorithm. To be fully instantiated, we need:

- A definition of the probability distributions,
- An algorithm for the E step
- An algorithm (or an analytical solution) for the M step
- Return final $\theta$

This step needs can be solved either analytically for simple problems or more often algorithmically.
Intuition

We want to maximize this function:

$$\log \text{likelihood}(D \mid \theta) = \sum_{i} \log \sum_{y \in \mathcal{Y}(x_i)} P_{\theta}(x_i, y)$$
Intuition

Start with a guess

\[
\log \text{likelihood}(D \mid \theta) = \sum_i \log \sum_{y \in y(x_i)} P_{\theta}(x_i, y)
\]
Intuition

Construct the expected log-likelihood function using the current guess and maximize it instead.

\[
\log \text{likelihood}(D \mid \theta) = \sum_i \log \sum_{y \in y(x_i)} P_{\theta}(x_i, y)
\]

\[
\theta^{t+1} \leftarrow \arg\max_{\theta} \sum_i E_{y \sim Q_t} [\log P_{\theta}(x_i, y)]
\]
Intuition

$$\text{log likelihood}(D \mid \theta) = \sum_i \log \sum_{y \in y(x_i)} P_{\theta}(x_i, y)$$

$$\theta^{t+1} \leftarrow \arg\max_{\theta} \sum_i E_{y \sim Q_t} [\log P_{\theta}(x_i, y)]$$

New set of parameters
Intuition

Construct the expected log-likelihood function using the current parameters and maximize it instead

\[
\log \text{likelihood}(D \mid \theta) = \sum_i \log \sum_{y \in y(x_i)} P_{\theta}(x_i, y)
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\log \text{likelihood}(D \mid \theta) = \sum_i \log \sum_{y \in y(x_i)} P_{\theta}(x_i, y)
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\[
\theta^{t+1} \leftarrow \arg\max_{\theta} \sum_i E_{y \sim Q_i^t} [\log P_{\theta}(x_i, y)]
\]
Intuition

\[ \text{log likelihood}(D \mid \theta) = \sum_i \log \sum_{y \in \mathcal{Y}(x_i)} P_\theta(x_i, y) \]

\[ \theta^{t+1} \leftarrow \arg\max_{\theta} \sum_i E_{y \sim Q_i^t}[\log P_\theta(x_i, y)] \]

New set of parameters
Intuition

Construct the expected log-likelihood function using the current parameters and maximize it instead to get new set of parameters.

\[
\text{log likelihood}(D \mid \theta) = \sum_i \log \sum_{y \in y(x_i)} P_{\theta}(x_i, y)
\]

\[
\theta^{t+1} \leftarrow \arg \max_{\theta} \sum_i E_{y \sim q_i^t} [\log P_{\theta}(x_i, y)]
\]
Intuition

1. Our initial guess matters, we could have landed on another local maximum as well. But we will always end up at one of the local maxima.

2. We are replacing our “difficult” optimization problems with a sequence of “easy” ones.

Mathematical equation:

$$\log \text{likelihood}(D \mid \theta) = \sum_i \log \sum_{y \in y(x_i)} P_{\theta}(x_i, y)$$

Update rule:

$$\theta^{t+1} \leftarrow \arg\max_\theta \sum_i E_{y \sim Q_i^t}[\log P_{\theta}(x_i, y)]$$
Comments about EM

• Will converge to a local maximum of the log-likelihood
  – Different initializations can give us different final estimates of probabilities

• How many iterations
  – Till convergence. Keep track of expected log likelihood across iterations and if the change is smaller than some \( \epsilon \) then stop

• What we need to specify the learning algorithm
  – A task-specific definition of the probabilities
  – A way to solve the maximization (the M-step)

• Gives us a general template for building models when we don’t have fully labeled data
This lecture

• What is weak supervision?

• The expectation maximization algorithm
  – A general template for learning with weak supervision

• Learning with latent variables

• Learning with constraints
Learning with latent variables

Given a dataset with examples $x_i$ that are labeled with a partial structure $y_i$

- The partial structure could be empty, giving us the fully unsupervised setting
- Or it could have labeled some parts of the structure for each example
Learning with latent variables

Given a dataset with examples \(\mathbf{x}_i\) that are labeled with a partial structure \(\mathbf{y}_i\)

- Initialize the parameters \(\theta^0\) randomly

- Iterate for \(t = 1, 2, \ldots\) :
  - Use the current model to “complete” each example
  - \(\theta^{t+1} \leftarrow\) Train model using the newly completed examples

- Return the final set of parameters
Learning with latent variables

Given a dataset with examples $x_i$ that are labeled with a partial structure $y_i$

- Initialize the parameters $\theta^0$ randomly
- Iterate for $t = 1, 2, \ldots$ :
  - Use the current model to “complete” each example
  - $\theta^{t+1} \leftarrow$ Train model using the newly completed examples
- Return the final set of parameters

This step requires some type of inference
Learning with latent variables

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- Return the final set of parameters

This step requires some type of inference

The completed examples could involve a hard decision (i.e. a structure), or a soft decision (i.e. a probability for each label)
Specific instances of this idea

• The EM algorithm for hidden Markov Models: The Baum Welch algorithm

• When the learner in the model update step is the structural SVM, we get the latent structured SVM
  – (with some caveats about the cost of misclassification, see Yu & Joachims 2009 for details)

• We can instantiate an EM algorithm for CRF style models as well
A simple example

**Setting:** Inputs are denoted by $x_i$, and each input is associated with a collection of outputs each of which can be $A$, $B$ or $C$.

A fully labeled example may look like $\begin{array}{c} x \end{array} \begin{array}{cccc} B & B & C & A \end{array}$
A simple example

**Setting:** Inputs are denoted by $x_i$, and each input is associated with a collection of outputs each of which can be $A$, $B$ or $C$.

A fully labeled example may look like $x \quad B \quad B \quad C \quad A$

Suppose we are assuming this model:
A simple example

**Setting:** Inputs are denoted by $x_i$, and each input is associated with a collection of outputs each of which can be $A$, $B$, or $C$.

A fully labeled example may look like $x \begin{bmatrix} B \ B \ C \ A \end{bmatrix}$

Suppose we are assuming this model:

We have three training examples:

- $x_1 \begin{bmatrix} _ & A & _ \end{bmatrix}$
- $x_2 \begin{bmatrix} B & _ & _ & C \end{bmatrix}$
- $x_3 \begin{bmatrix} A & _ \end{bmatrix}$
A simple example

**Setting:** Inputs are denoted by $x_i$, and each input is associated with a collection of outputs each of which can be $A$, $B$ or $C$.

A fully labeled example may look like $x$ \begin{tabular}{llll} B & B & C & A \\ \end{tabular}

Suppose we are assuming this model:

We have three training examples:

\begin{tabular}{l}
$x_1$ \begin{tabular}{ll} _ & A & _ \\ \end{tabular} \\
$x_2$ \begin{tabular}{llll} B & _ & _ & C \\ \end{tabular} \\
$x_3$ \begin{tabular}{ll} A & _ \\ \end{tabular} \\
\end{tabular}

What can we do with this data?
A simple example

**Setting:** Inputs are denoted by $x_i$, and each input is associated with a collection of outputs each of which can be $A$, $B$ or $C$.

A fully labeled example may look like $x \quad B \quad B \quad C \quad A$

Suppose we are assuming this model:

We have three training examples:

$\begin{array}{ccc}
  x_1 & \_ & A & \_ \\
  x_2 & B & \_ & \_ & C \\
  x_3 & A & \_ & \\
\end{array}$

What can we do with this data? If we had the parameters of the model, we can predict the missing labels.
A simple example

Setting: Inputs are denoted by $x_i$, and each input is associated with a collection of outputs each of which can be $A$, $B$ or $C$.

We have three training examples:

If we had the parameters of the model, we can predict the missing labels

The annotated part of the output for this example

The unannotated (i.e. hidden) part of the output for this example
**Setting**: Inputs are denoted by $x_i$, and each input is associated with a collection of outputs each of which can be $A$, $B$ or $C$.

We have three training examples:

If we had the parameters of the model, we can predict the missing labels.

Suppose the current model was denoted by $w$ and the corresponding scoring function as per the factor graph is $score(x, y, h, w)$.

The annotated part of the output for this example

The unannotated (i.e. hidden) part of the output for this example
A simple example

**Setting:** Inputs are denoted by $x_i$, and each input is associated with a collection of outputs each of which can be $A$, $B$ or $C$.

We have three training examples:

```
| x_1   | A   |
```

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The unannotated (i.e. hidden) part of the output for this example

We can ask: What is the best value for $h_1$ for this example as per the current model?
**A simple example**

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The unannotated (i.e. hidden) part of the output for this example

We can ask: What is the best value for $h_1$ for this example as per the current model?

$$h_1^* = \arg\max_h score(x_1, y_1, h, w)$$
A simple example

**Setting:** Inputs are denoted by $x_i$, and each input is associated with a collection of outputs each of which can be $A$, $B$ or $C$.

We have three training examples:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>A</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>_</td>
<td>A</td>
<td>_</td>
</tr>
<tr>
<td>$x_2$</td>
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<td>_</td>
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</tr>
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<td>_</td>
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<tr>
<td>$x_3$</td>
<td>A</td>
<td>_</td>
</tr>
</tbody>
</table>

If we had the parameters of the model, we can predict the missing labels.

Suppose the current model was denoted by $\mathbf{w}$ and the corresponding scoring function as per the factor graph is $score(x, y, h, \mathbf{w})$.

We can ask: What is the best value for $h$ for each example as per the current model?

$$h^*_i = \arg \max_h score(x_i, y_i, h, \mathbf{w})$$

Perform inference to find best estimates for hidden variables.
A simple example

Setting: Inputs are denoted by $x_i$, and each input is associated with a collection of outputs each of which can be $A$, $B$ or $C$.

We have three training examples:

If we had the parameters of the model, we can predict the missing labels

Suppose the current model was denoted by $\mathbf{w}$ and the corresponding scoring function as per the factor graph is $score(x, y, h, w)$

Complete the missing labels using $\mathbf{w}$
**A simple example**

**Setting:** Inputs are denoted by $x_i$, and each input is associated with a collection of outputs each of which can be $A$, $B$ or $C$.

We have three training examples:

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<td>A</td>
<td>_</td>
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If we had the parameters of the model, we can predict the missing labels.

Suppose the current model was denoted by $w$ and the corresponding scoring function as per the factor graph is $score(x, y, h, w)$.

Complete the missing labels using $w$.

<table>
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<th></th>
<th>B</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
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<td>B</td>
<td>A</td>
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</tr>
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<td>A</td>
<td></td>
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Use this newly completed data to train new update for $w$.

Use any loss function.
A simple example

Setting: Inputs are denoted by $x_i$, and each input is associated with a collection of outputs each of which can be $A$, $B$ or $C$.

We have three training examples:

- $x_1$: 
  - $A$
  - 

- $x_2$: $B$
  - 
  - $C$

- $x_3$: 
  - $A$

If we had the parameters of the model, we can predict the missing labels.

Suppose the current model was denoted by $w$ and the corresponding scoring function as per the factor graph is $score(x, y, h, w)$.

Complete the missing labels using $w$:

- $x_1$: $B$
  - $A$
  - $B$

- $x_2$: $B$
  - $A$
  - $C$
  - $C$

Use this newly completed data to train new update for $w$.

Use any loss function.
This lecture

• What is weak supervision?

• The expectation maximization algorithm
  – A general template for learning with weak supervision

• Learning with latent variables

• Learning with constraints
Constraint Driven Learning

Suppose we have some constraints about the output structures, but don’t have any (or much) labeled data.

Examples:

• Every part of speech sequence should have a verb
• Every image patch that is recognized as a bicycle should have at least one patch that is recognized as a wheel
Constraint Driven Learning

Suppose we have some constraints about the output structures, but don’t have any (or much) labeled data

Examples:

• Every part of speech sequence should have a verb
• Every image patch that is recognized as a bicycle should have at least one patch that is recognized as a wheel

*How do we incorporate constraints into the learning process?*
Constraint Driven Learning

Given a dataset with examples $x_i$ that are labeled with a partial structure $y_i$

• Initialize the parameters $\theta^0$ randomly

• Iterate for $t = 1, 2, \ldots$:
  – Use the current model to “complete” each example
  – $\theta^{t+1} \leftarrow$ Train model using the newly completed examples

• Return the final set of parameters

This step requires some type of inference
Constraint Driven Learning

Given a dataset with examples \( x_i \) that are labeled with a partial structure \( y_i \)

• Initialize the parameters \( \theta^0 \) randomly

• Iterate for \( t = 1, 2, \ldots \) :
  – Use the current model to “complete” each example with constrained inference
  – \( \theta^{t+1} \leftarrow \) Train model using the newly completed examples

• Return the final set of parameters
Constraint Driven Learning

Given a dataset with examples $x_i$ that are labeled with a partial structure $y_i$

- Initialize the parameters $\theta^0$ randomly

- Iterate for $t = 1, 2, \ldots$ :
  - Use the current model to “complete” each example with constrained inference
  - $\theta^{t+1} \leftarrow$ Train model using the newly completed examples

- Return the final set of parameters

Variants of this idea exist. See [Chang et al, 2007, 2012]
Learning with constraints

*General idea: In the step where we expand (or complete) the partially labeled structures, use knowledge to guide the learner*

- Any of the inference algorithms we have seen can be used

- *Posterior regularization* Ganchev et al [2010] show that this idea applies for the case where we are not making a hard decision, but using the EM algorithm we have seen before

- Why do constraints help?
  - They restrict the search space for inference algorithms using knowledge
  - Constraints guide learning by making the learner explore the “better” parts of the parameter space
Final words

- Annotated data is expensive, more so for structures

- We can and should design learning algorithms that take advantage of any available supervision
  - Could be labeled, unlabeled or partially labeled examples
  - Could be knowledge

- The EM algorithm and its variants form a general schematic for designing such algorithms
  - Generally has one phase where we predict labels or distributions over labels for examples, and another where we use these predictions to improve the model

- Enforcing constraints during the prediction phase the can help reduce demand on labeled data