Decoding algorithms

THE
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OF UTAH

Where we are

We have seen different models for predicting sequences of tokens

- The underlying model could be a RNN or a transformer decoder

These autoregressive models

- Produce one token at a time
- The most recently predicted token is the input for the next step

The process of producing an output sequence involves chaining togther multiple decisions

This lecture: Decoding algorithms

How to predict a sequence

- The setup: An abstract auto-regressive model
- Decoding algorithms

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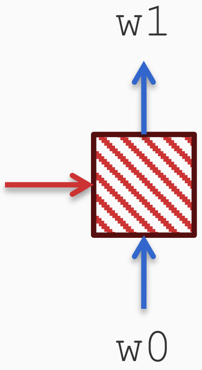
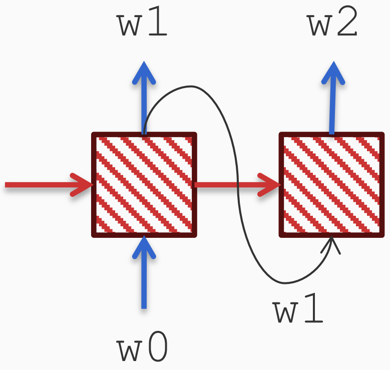


Recurrent neural network

The setup: Any autoregressive model



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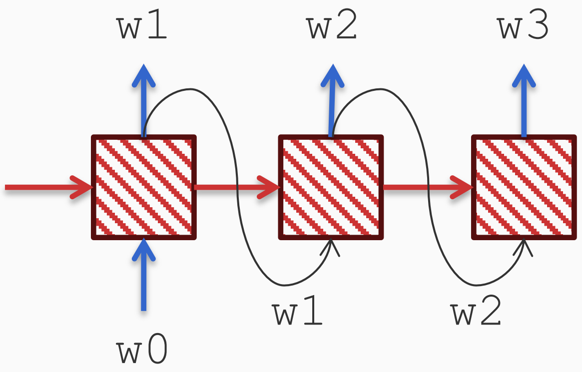

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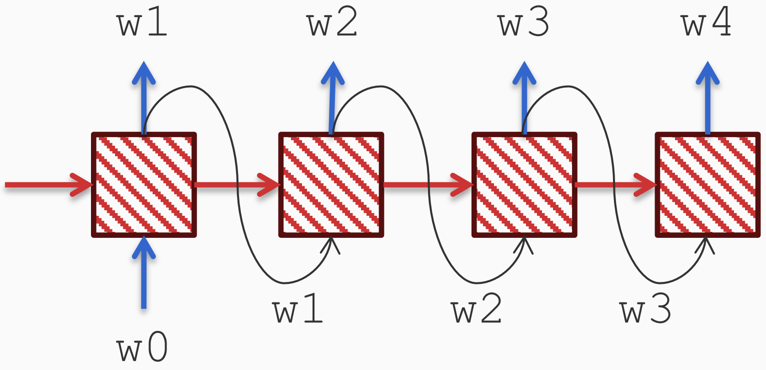

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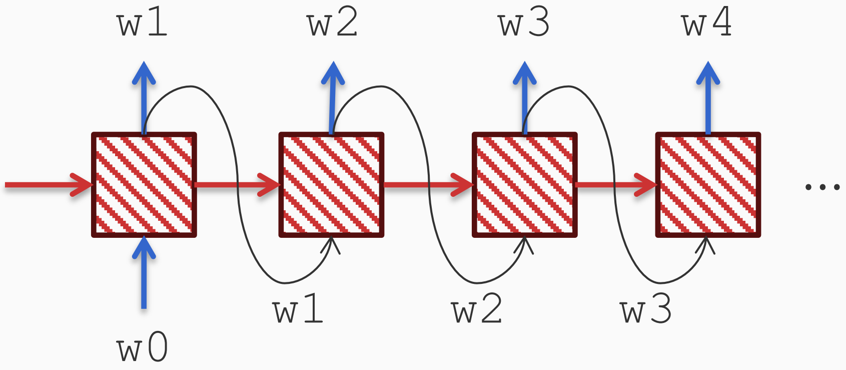

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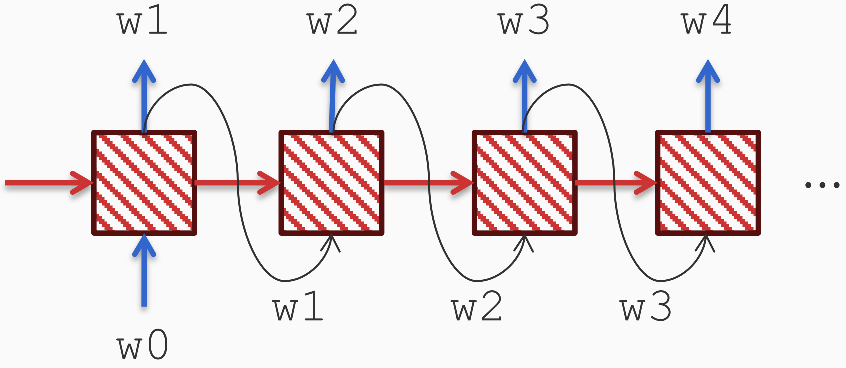

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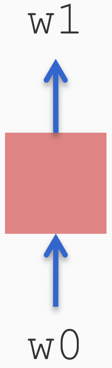

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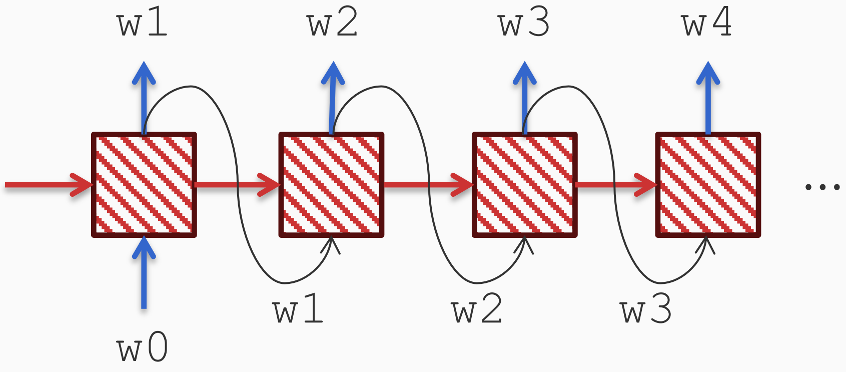

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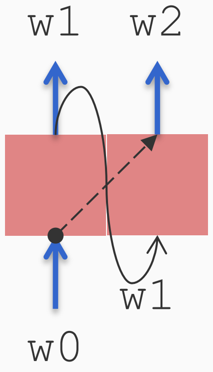

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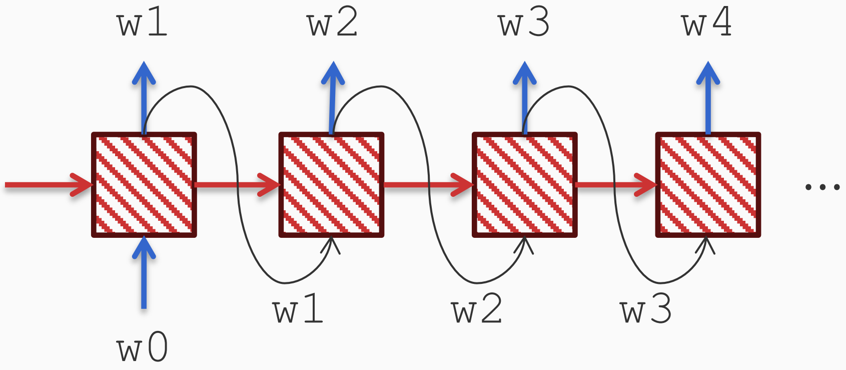

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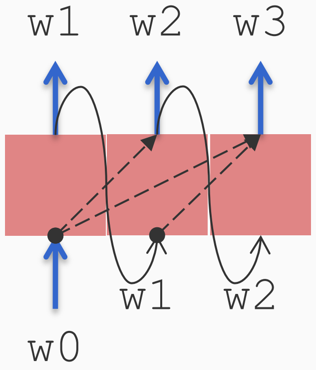

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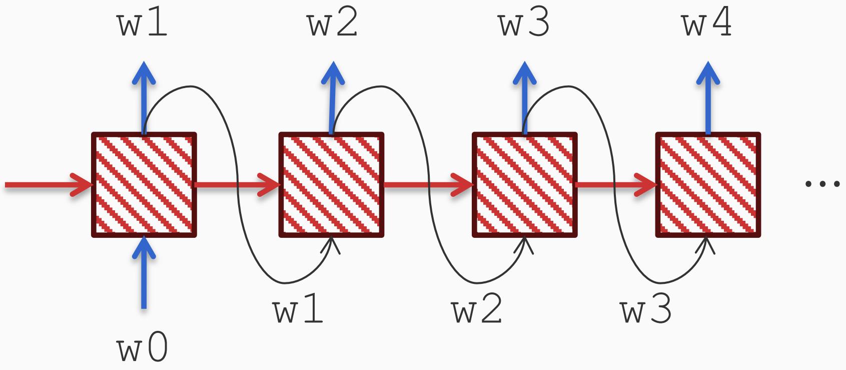

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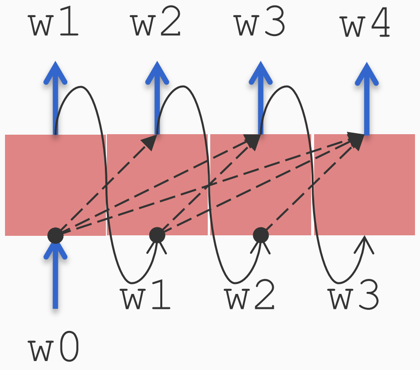

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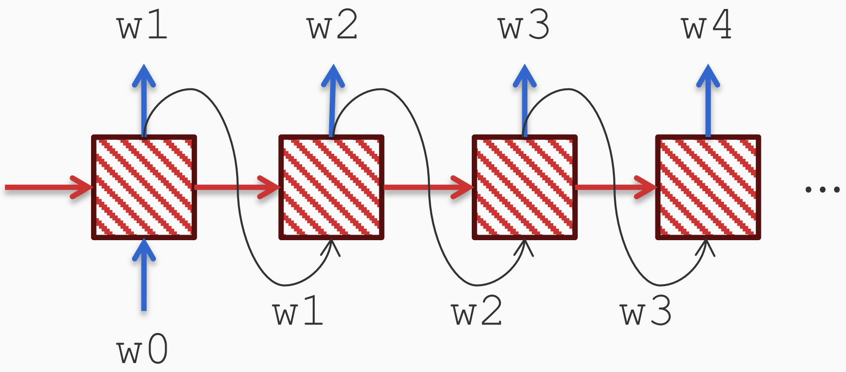

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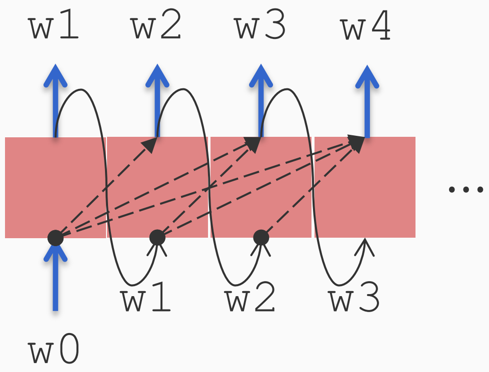

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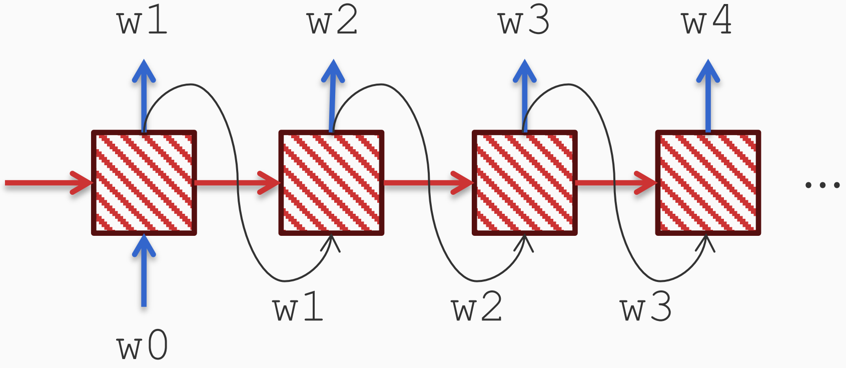

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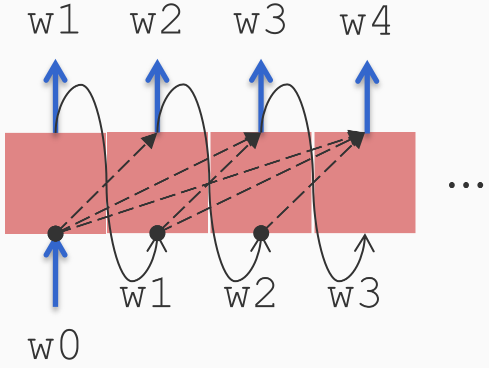
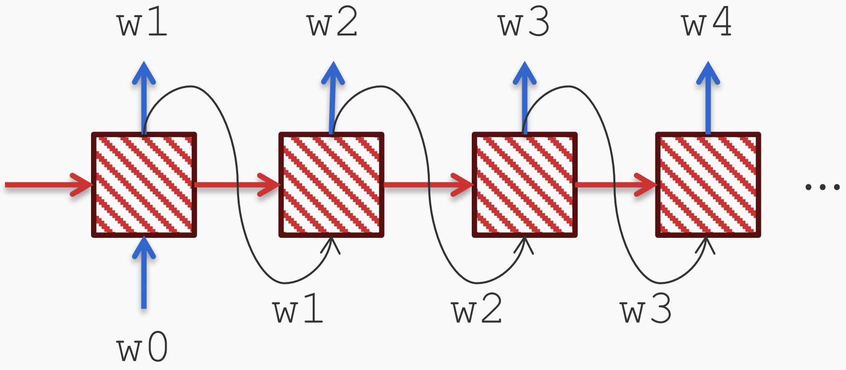


Transformer decoder

Both are ways to define probabilities for the next word given the sequence so far

The setup: Any autoregressive model

$$P(w_0 w_1 w_2 w_3 w_4 \cdots) = P(\text{token} \mid w_0) \cdot P(\text{token} \mid w_0 w_1) \cdot P(\text{token} \mid w_0 w_1 w_2) \cdot P(\text{token} \mid w_0 w_1 w_2 w_3) \cdots$$



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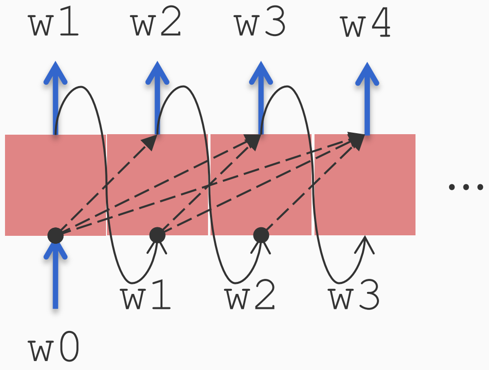

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The token `w1` is constructed using this distribution

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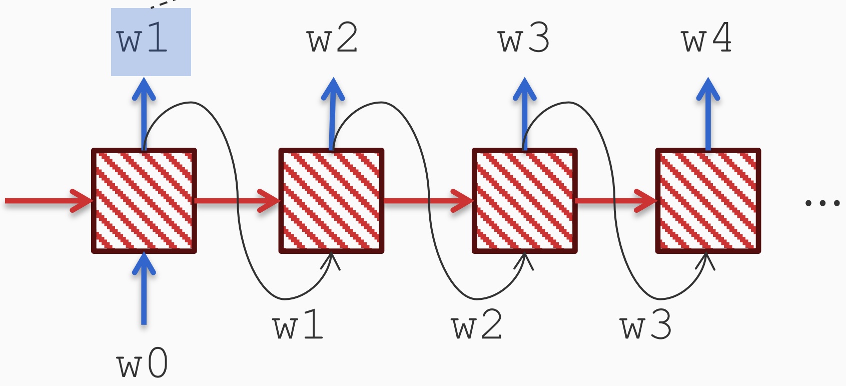

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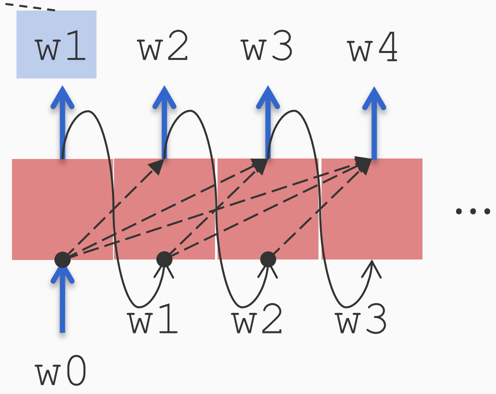


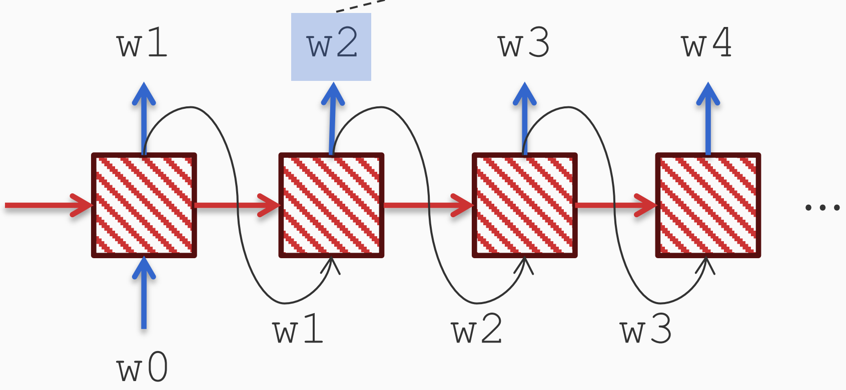
Transformer decoder

Both are ways to define probabilities for the next word given the sequence so far

The setup: Any autoregressive model

The token w2 is constructed
using this distribution

$$P(w_0 w_1 w_2 w_3 w_4 \cdots) = P(\text{token} \mid w_0) \cdot \boxed{P(\text{token} \mid w_0 w_1)} \cdot P(\text{token} \mid w_0 w_1 w_2) \cdot P(\text{token} \mid w_0 w_1 w_2 w_3) \cdots$$



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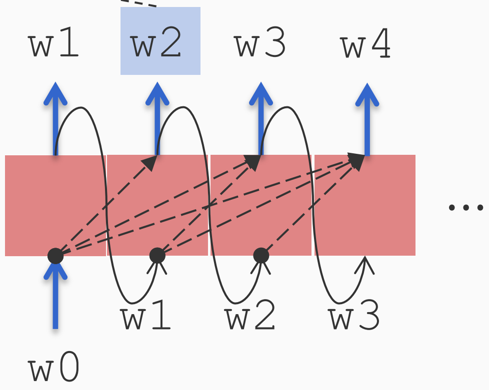

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The setup: Any autoregressive model

The token `w3` is constructed using this distribution

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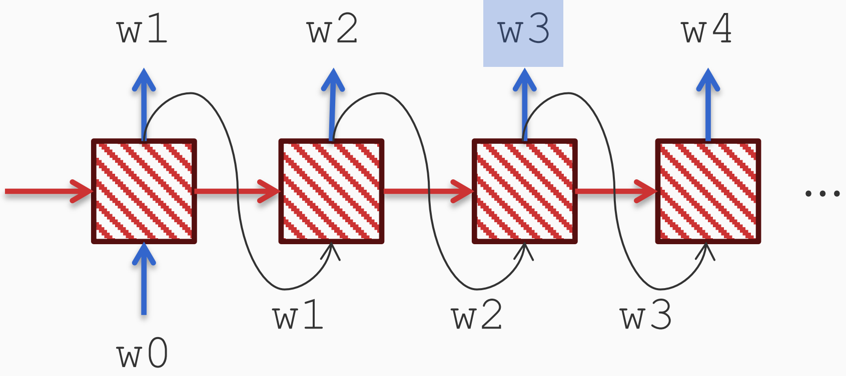

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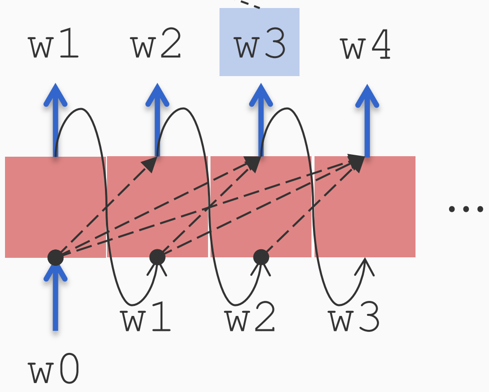

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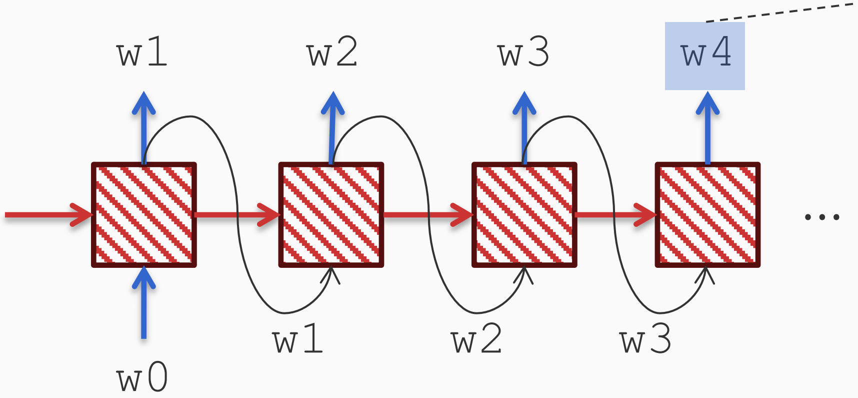
The token `w4` is constructed using this distribution

$$P(w_0 w_1 w_2 w_3 w_4 \cdots) = P(\text{token} \mid w_0) \cdot P(\text{token} \mid w_0 w_1) \cdot P(\text{token} \mid w_0 w_1 w_2) \cdot \boxed{P(\text{token} \mid w_0 w_1 w_2 w_3)} \cdots$$



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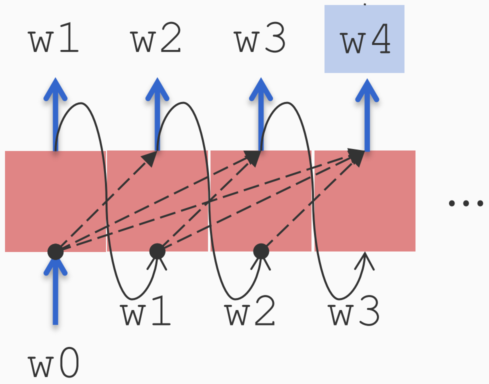

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Decoding: The algorithmic question

Given some method to create a probability distribution $P(\mathbf{token} \mid w_0 w_1 w_2 \cdots)$
how should we predict the “best” sequence?

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What does *best* mean? Ideas?

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What does *best* mean? Ideas?

Some notions of best when it comes to generating text

- Most probable
- Fast
- Does not repeat
- Diverse outputs
- ....

A toy example

Suppose our language model can pick from one of the following words at any step:

a, the, he, she, saw, him, her, apple

Some model predicts a conditional distribution given the words seen so far

Every token in the vocabulary is assigned a probability

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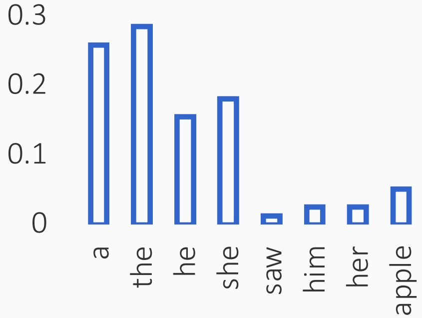

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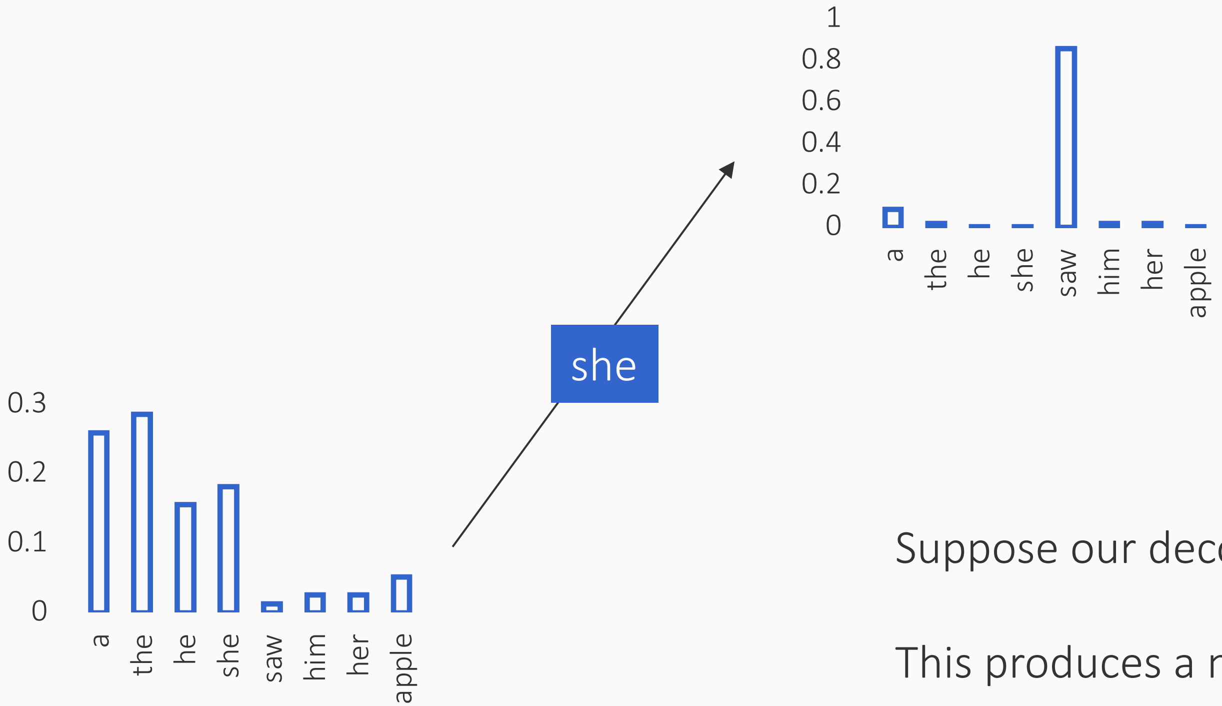

Suppose our decoder decides to pick the word "she"

This produces a new distribution over the next token

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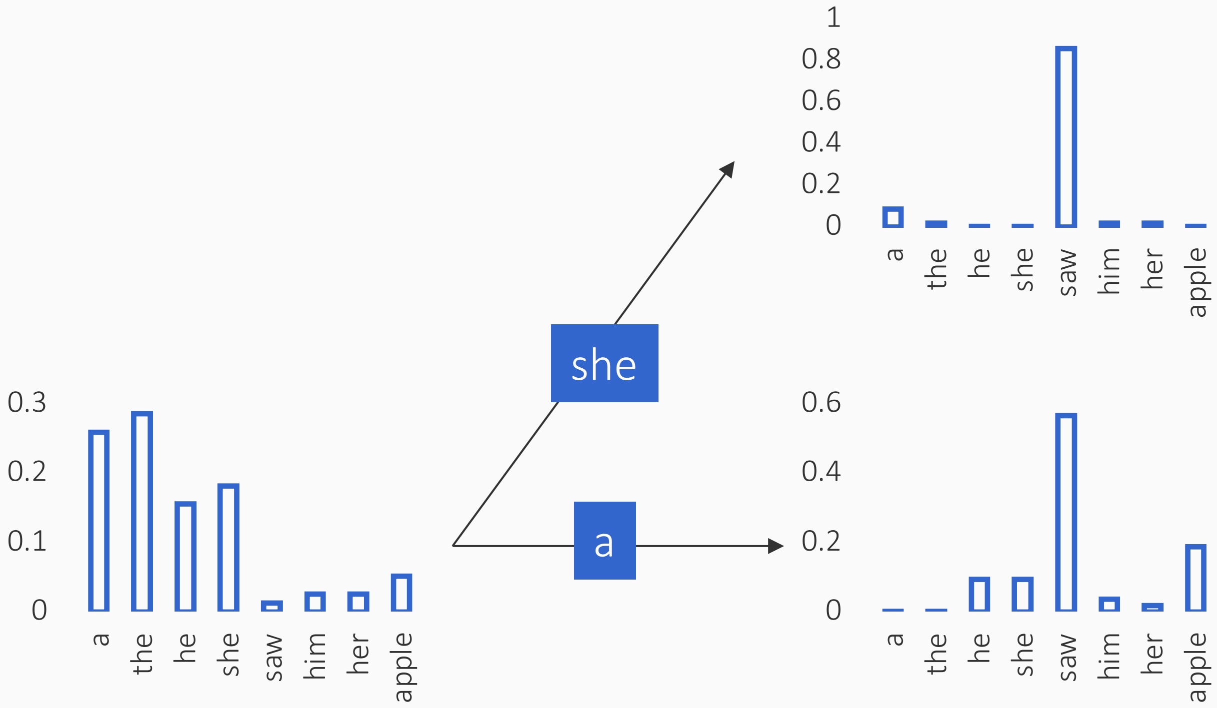

If it picked a different token, say “a”, then the next token distribution would be different.

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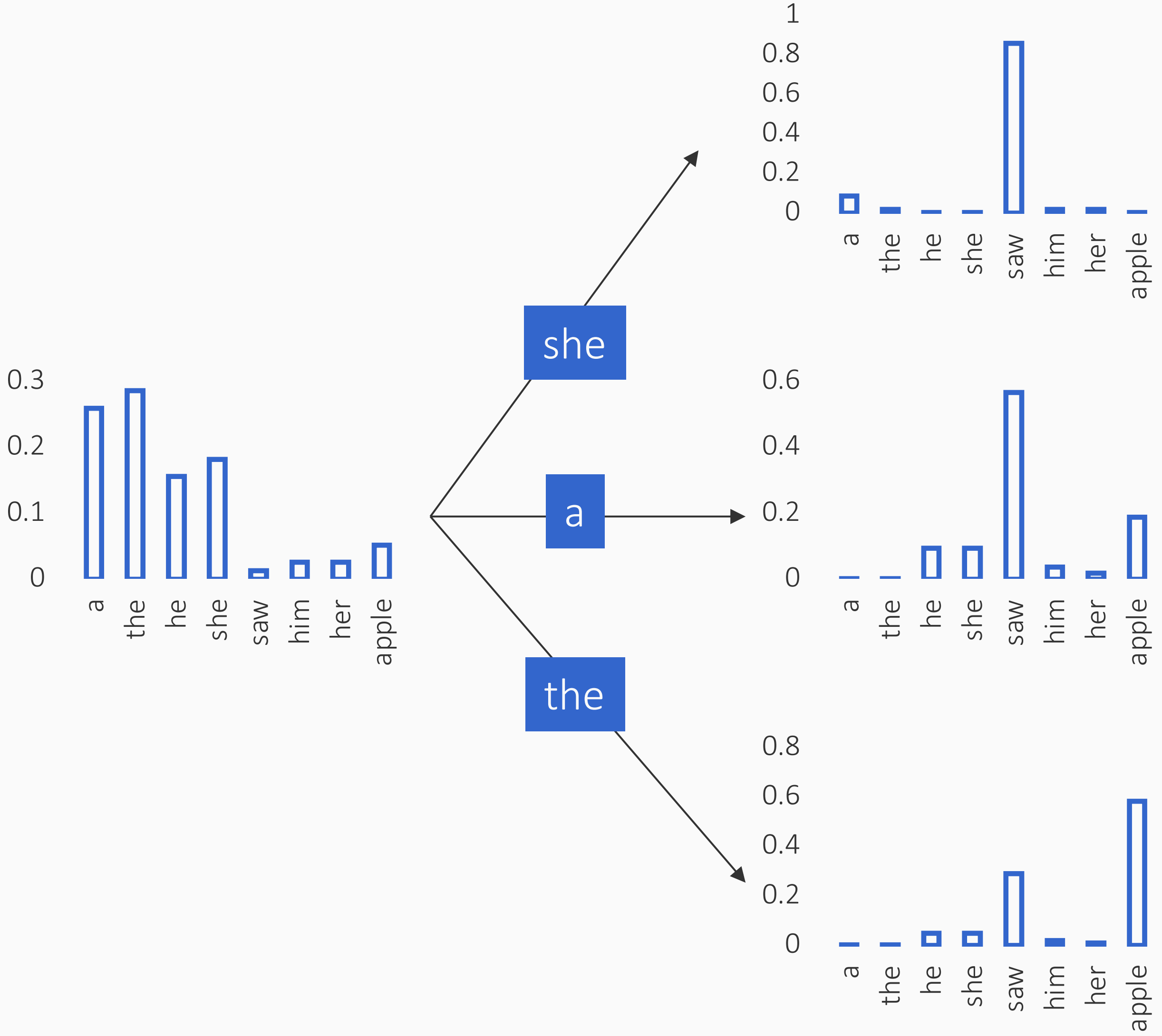

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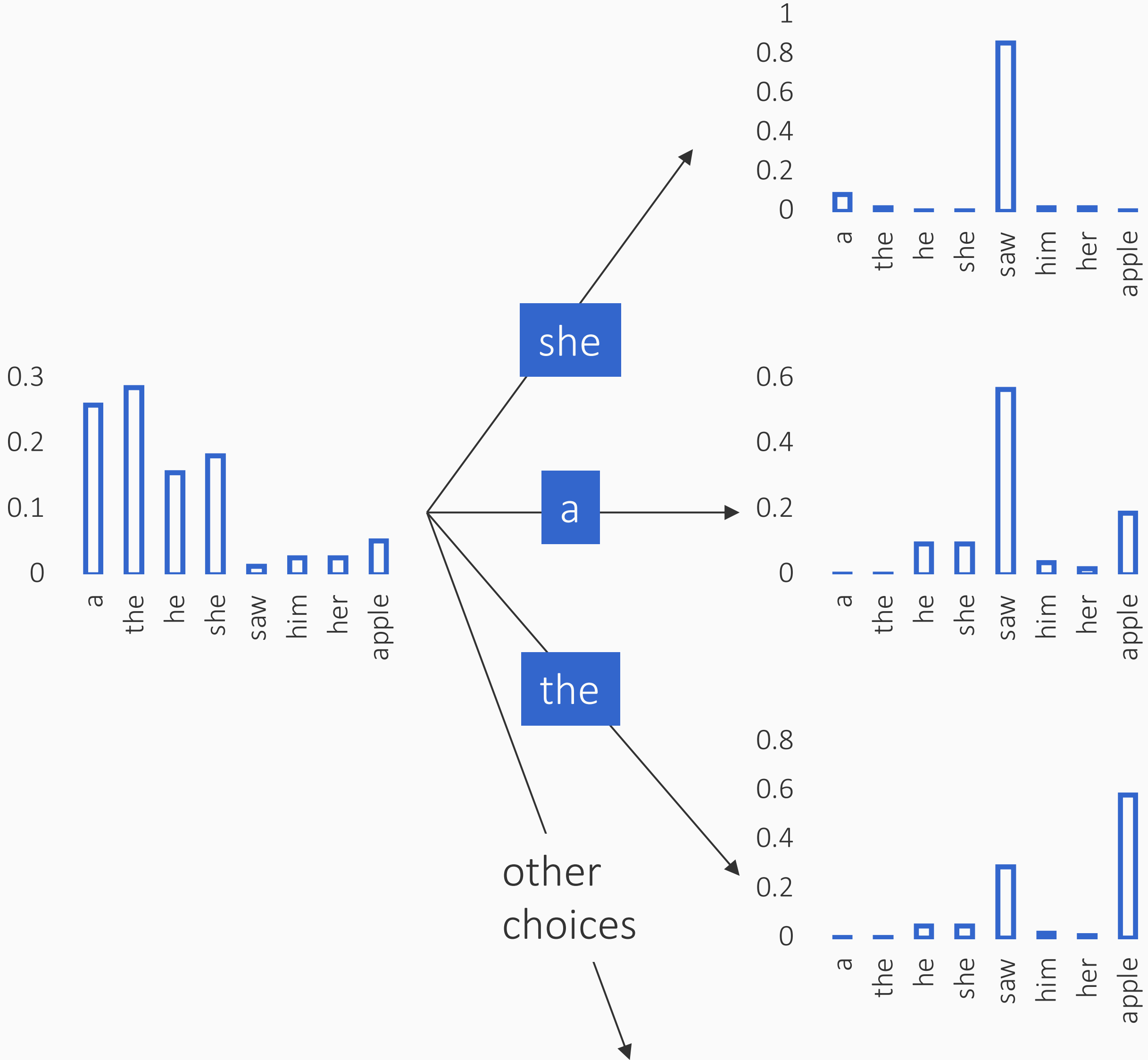

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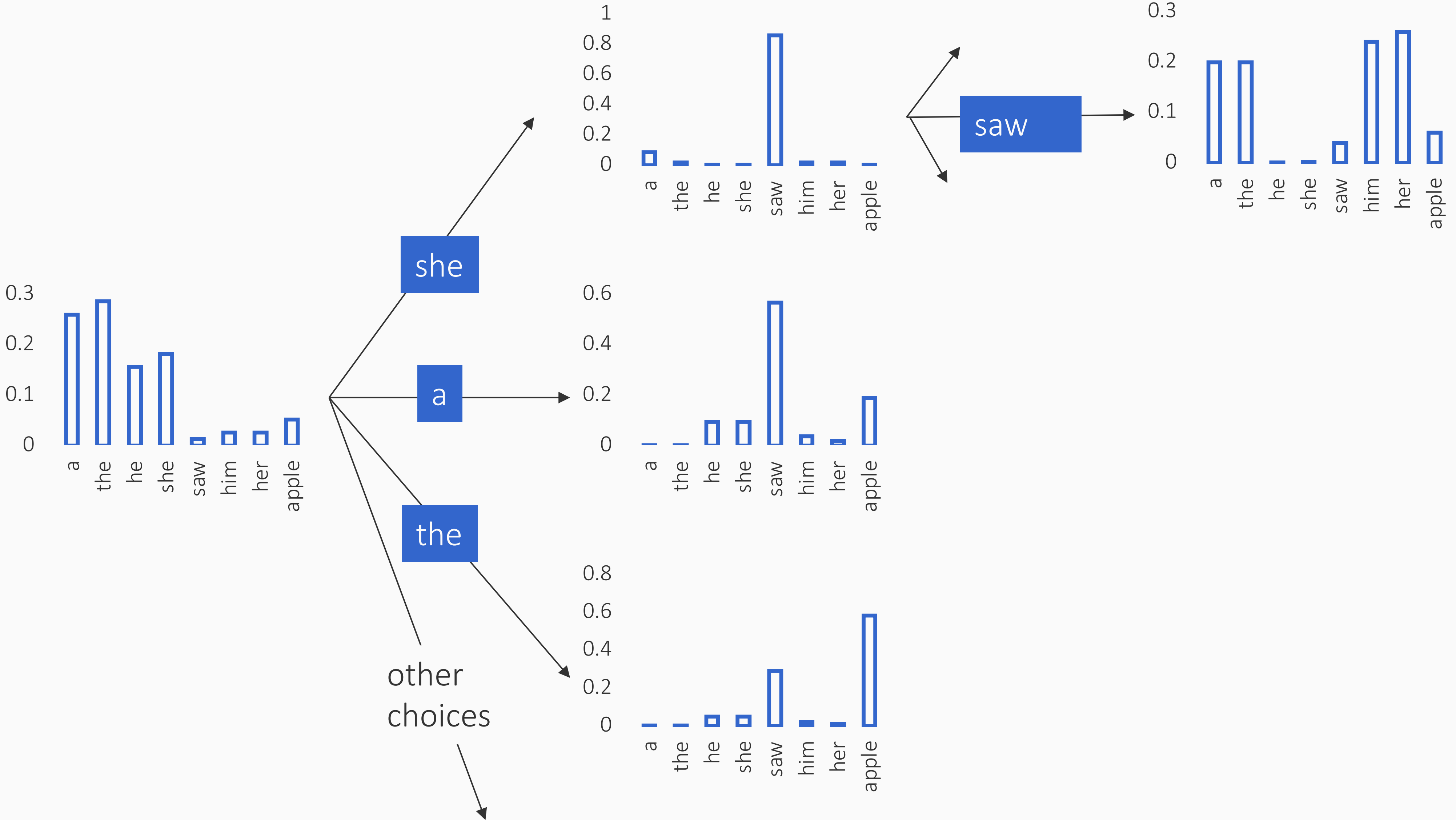

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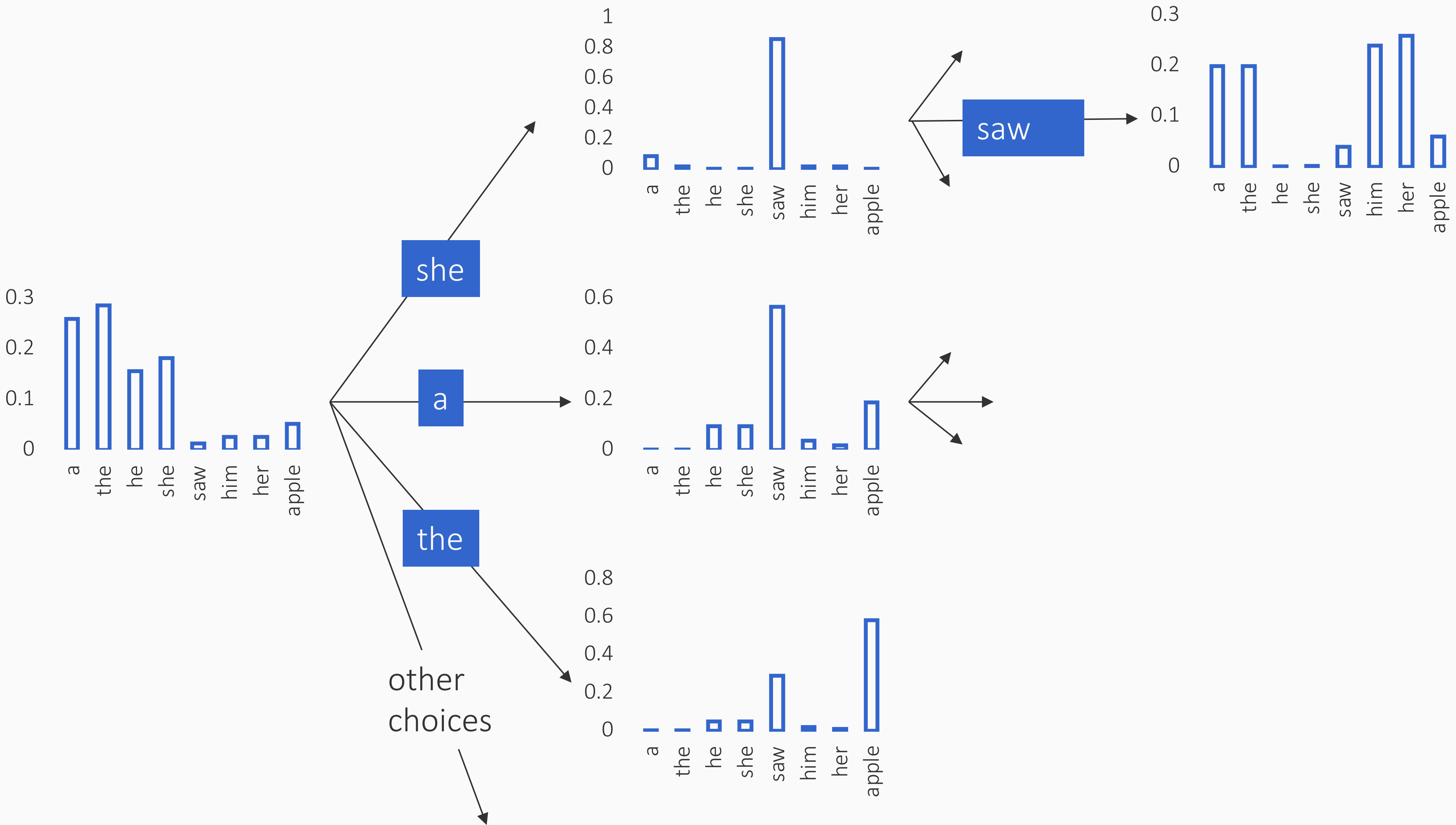

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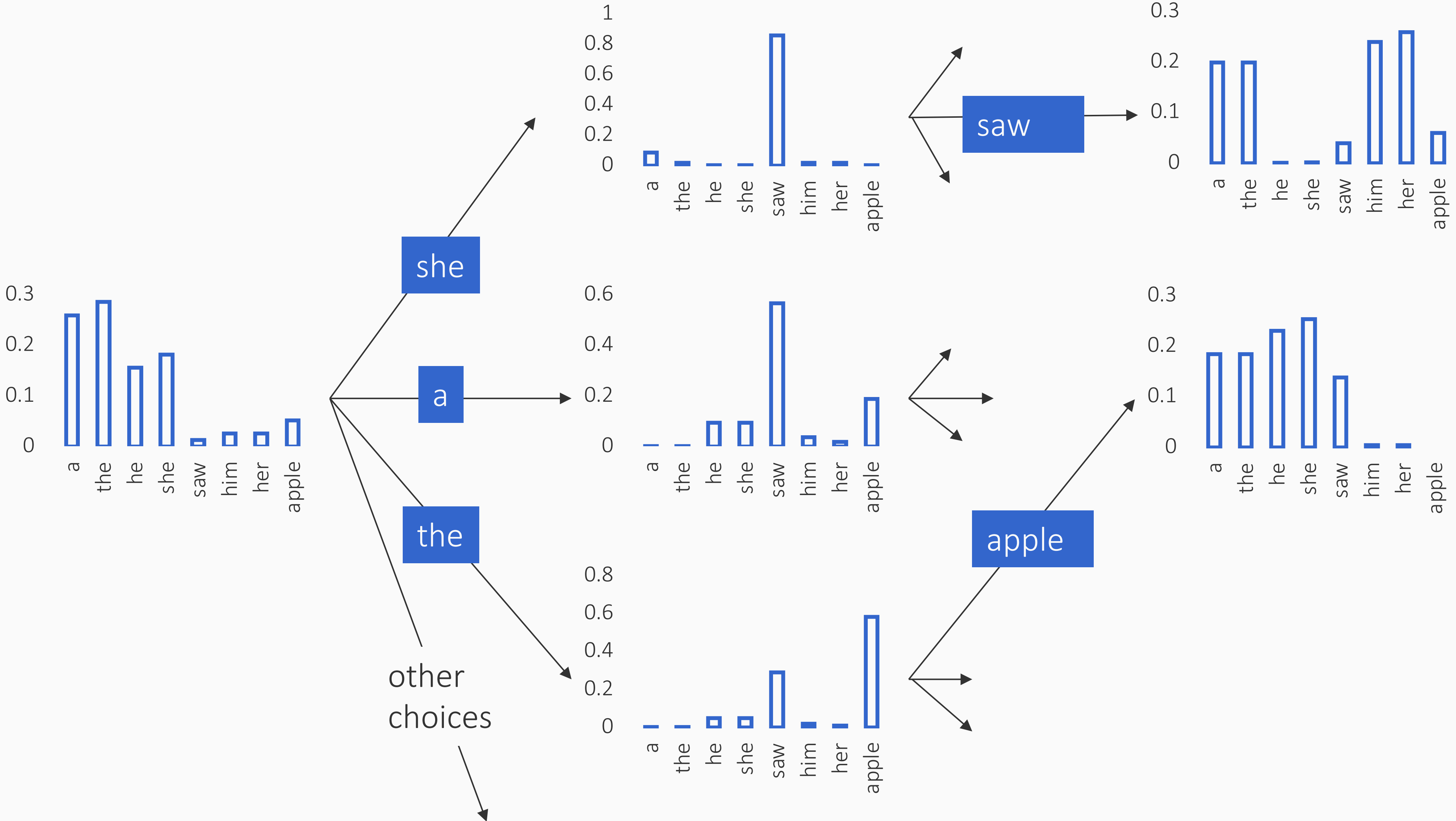

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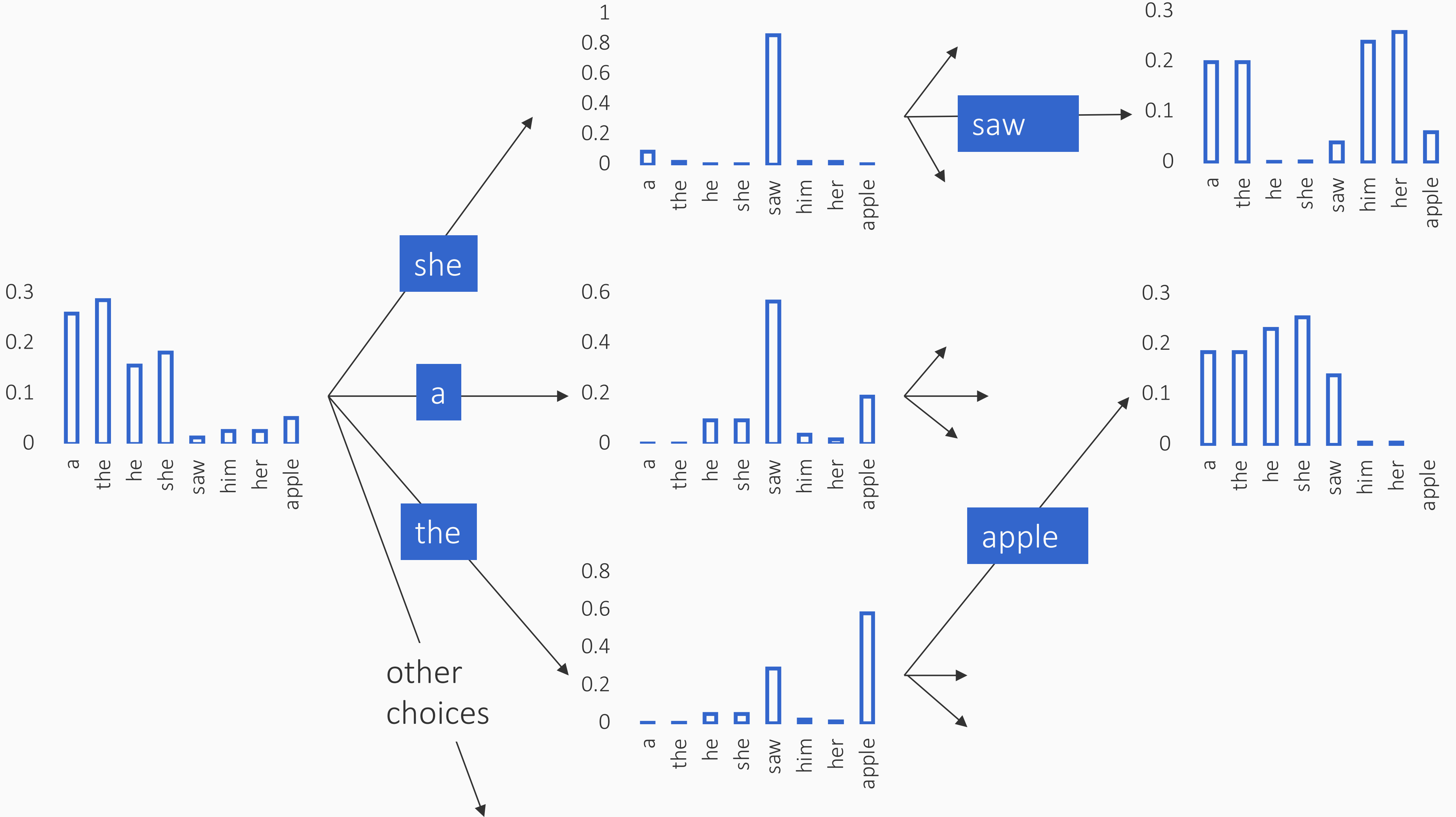


she

a

the

other choices

saw

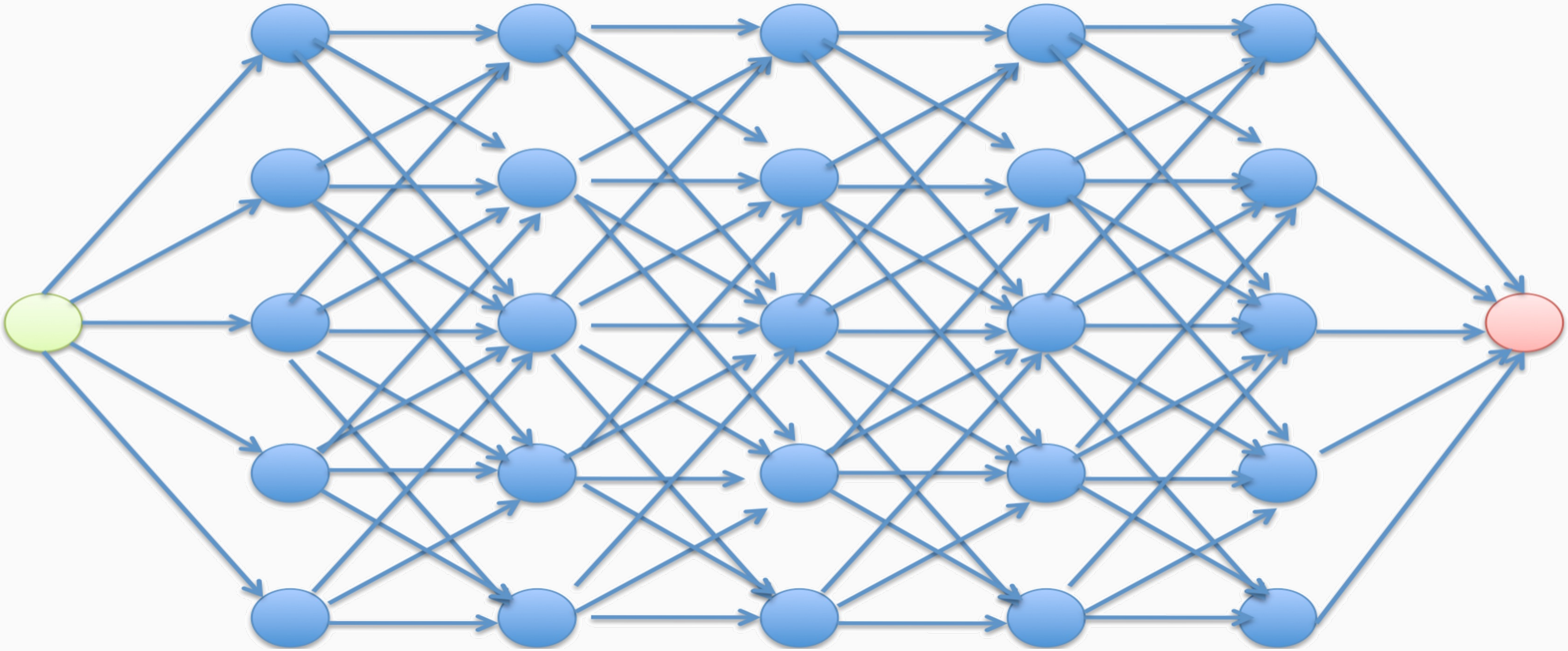
apple

And so on…

The search space for decoding

Each circle is a word that gets picked at a certain step

Goal: To find the "best" path in this graph



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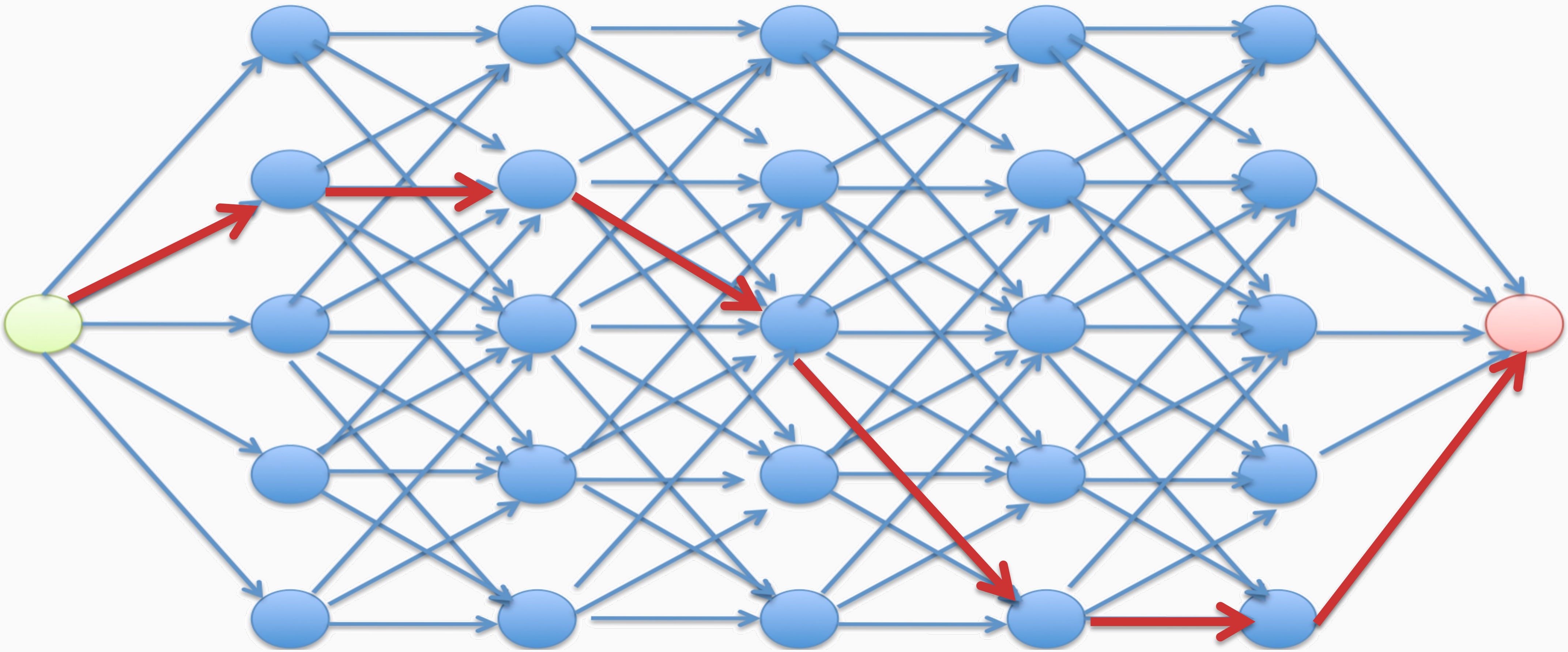

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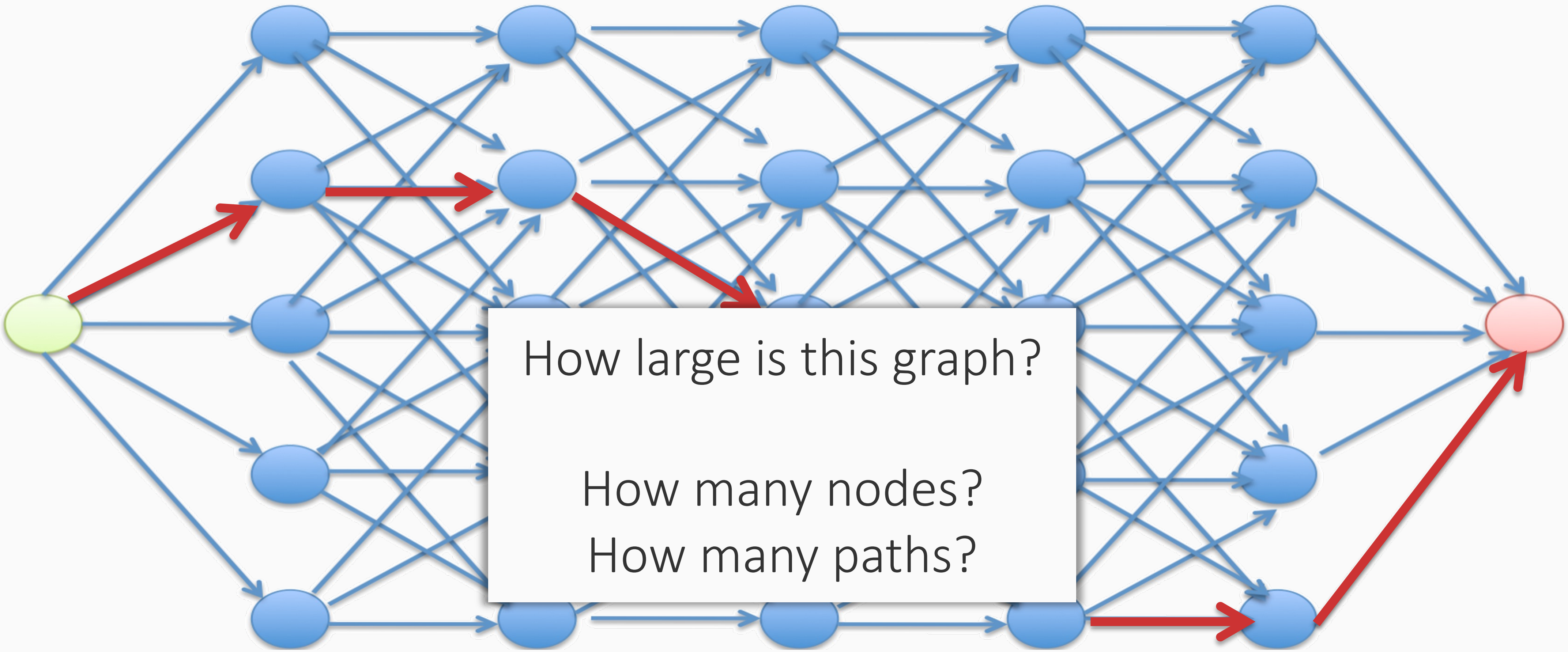

How large is this graph?

How many nodes?
How many paths?

Different decoding strategies exist

Search based decoding

Sampling based decoding

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Deterministic approaches that involves searching the space of sequences to select one

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- Greedy decoding
- Beam search

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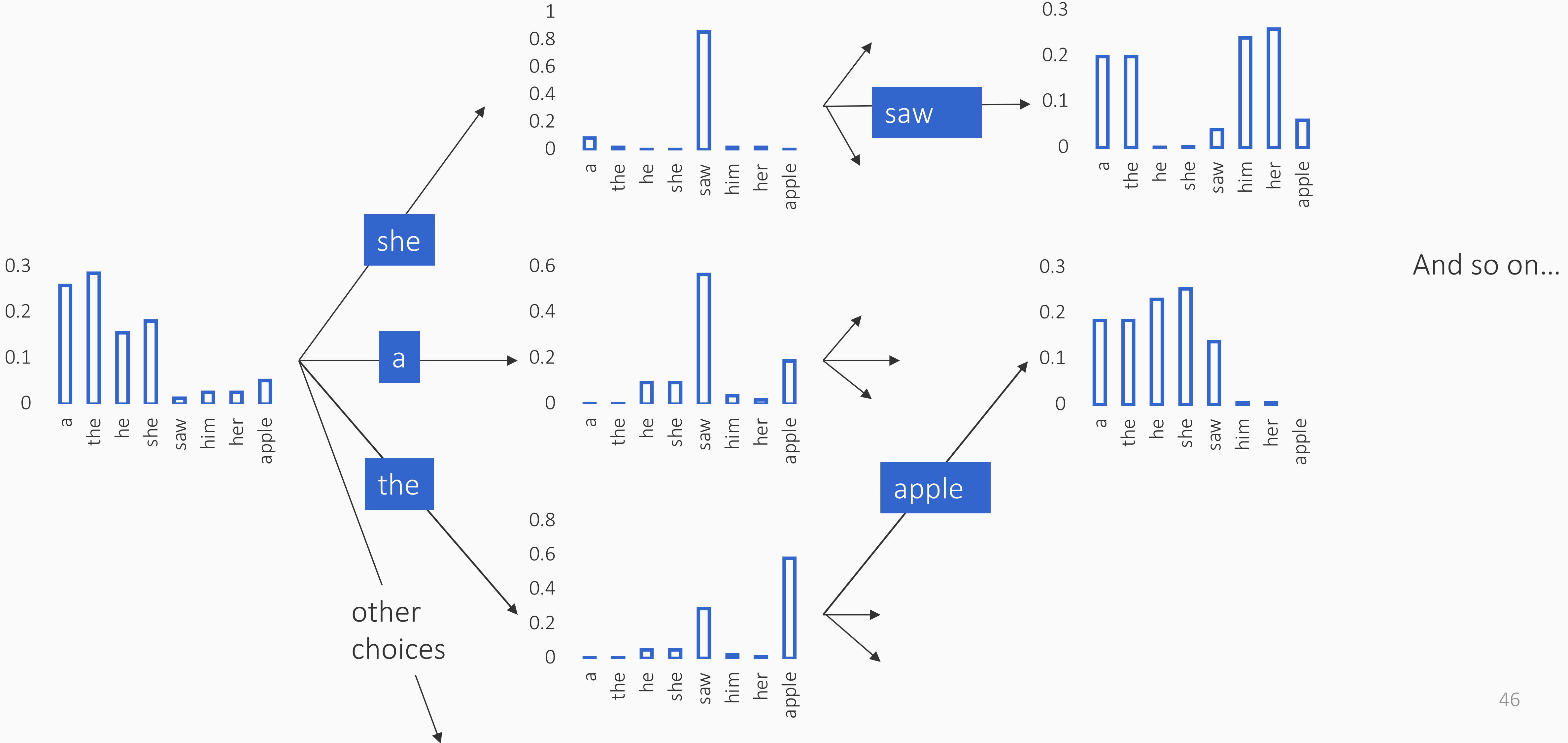
At each step, pick the token that has the highest probability

$$w_n = \underset{\{v \in \text{Vocabulary}\}}{\operatorname{argmax}} P\left(v \mid w_0 w_1 \cdots w_{\{n-1\}} \right)$$

Greedy decoding in our toy setting



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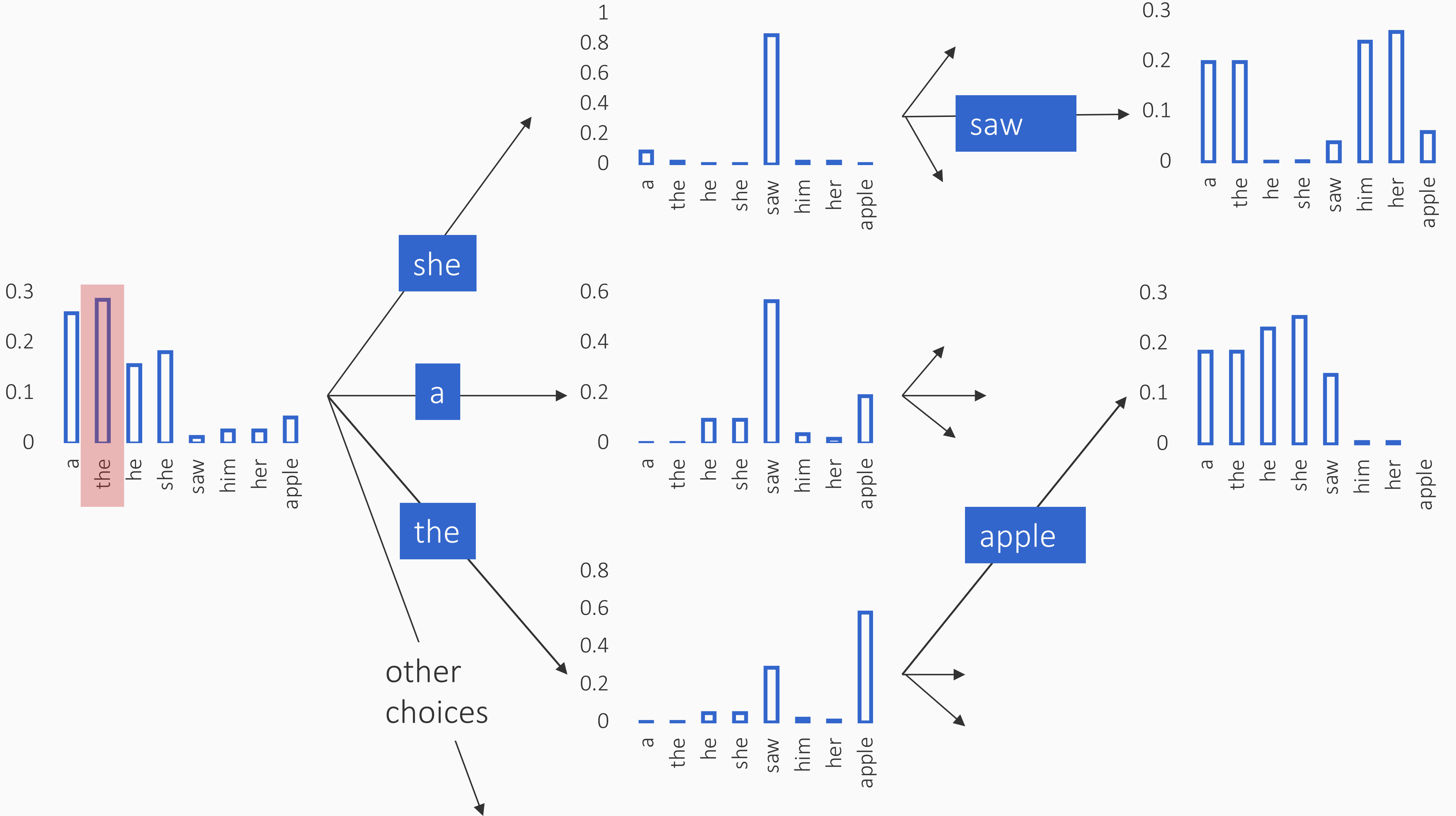

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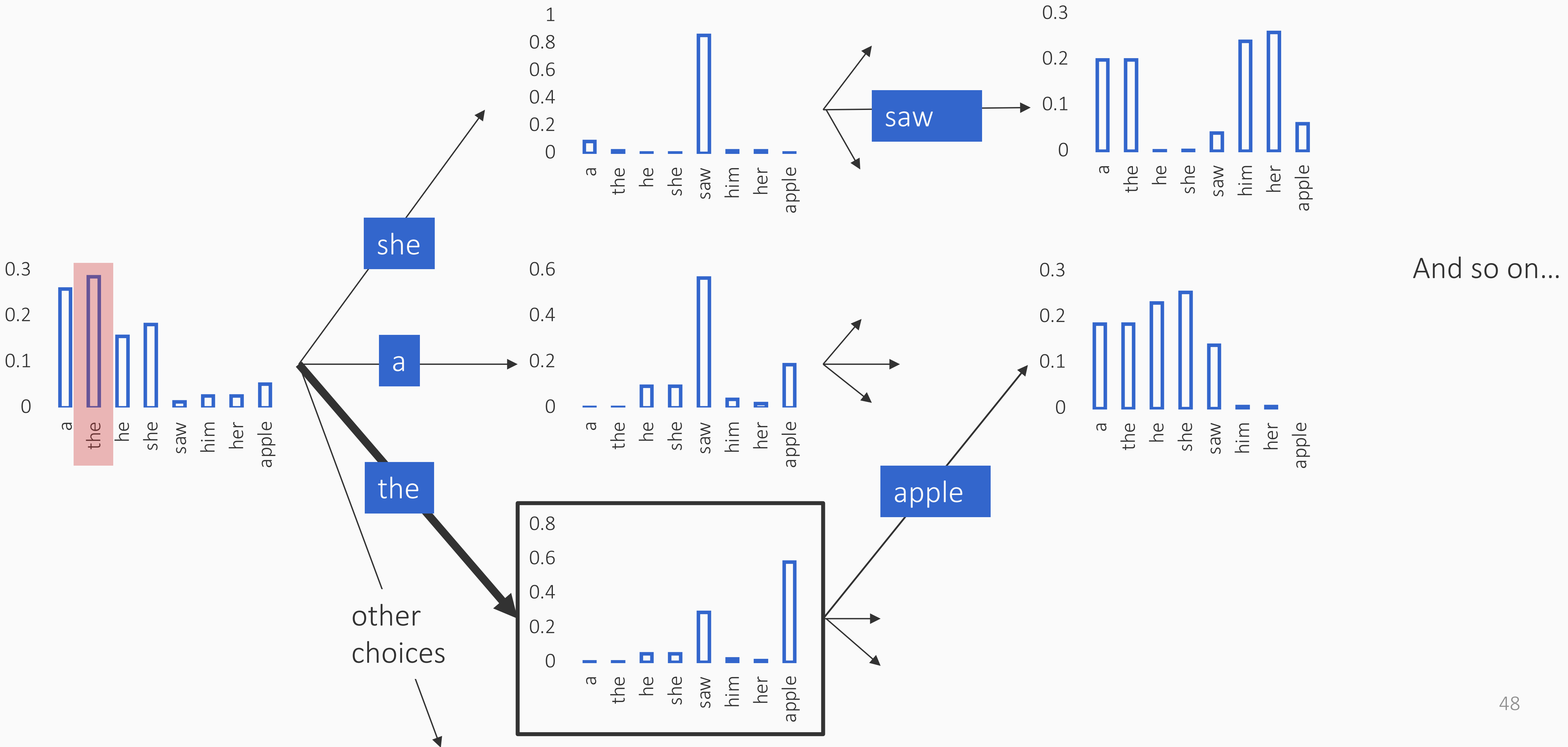

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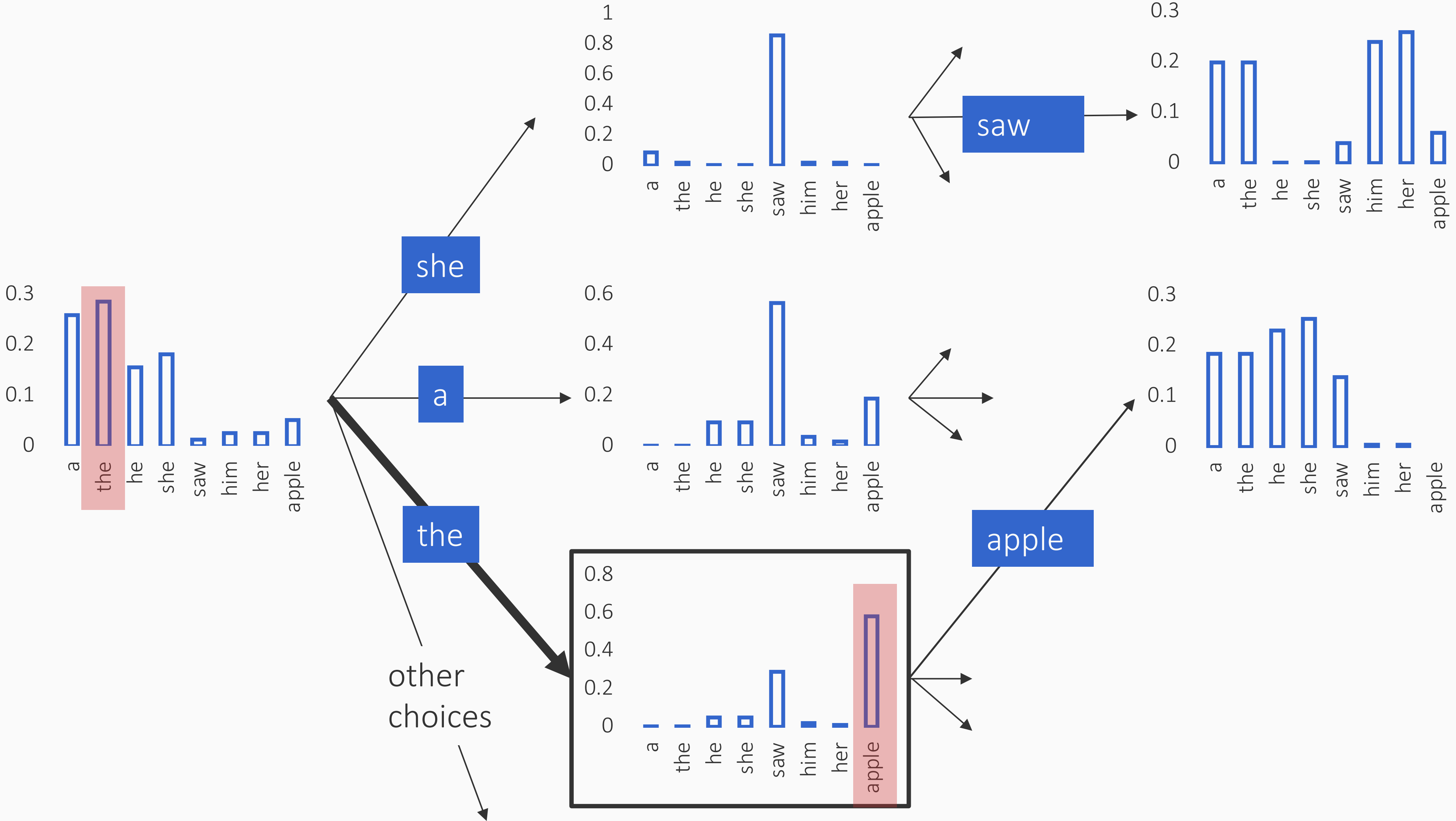

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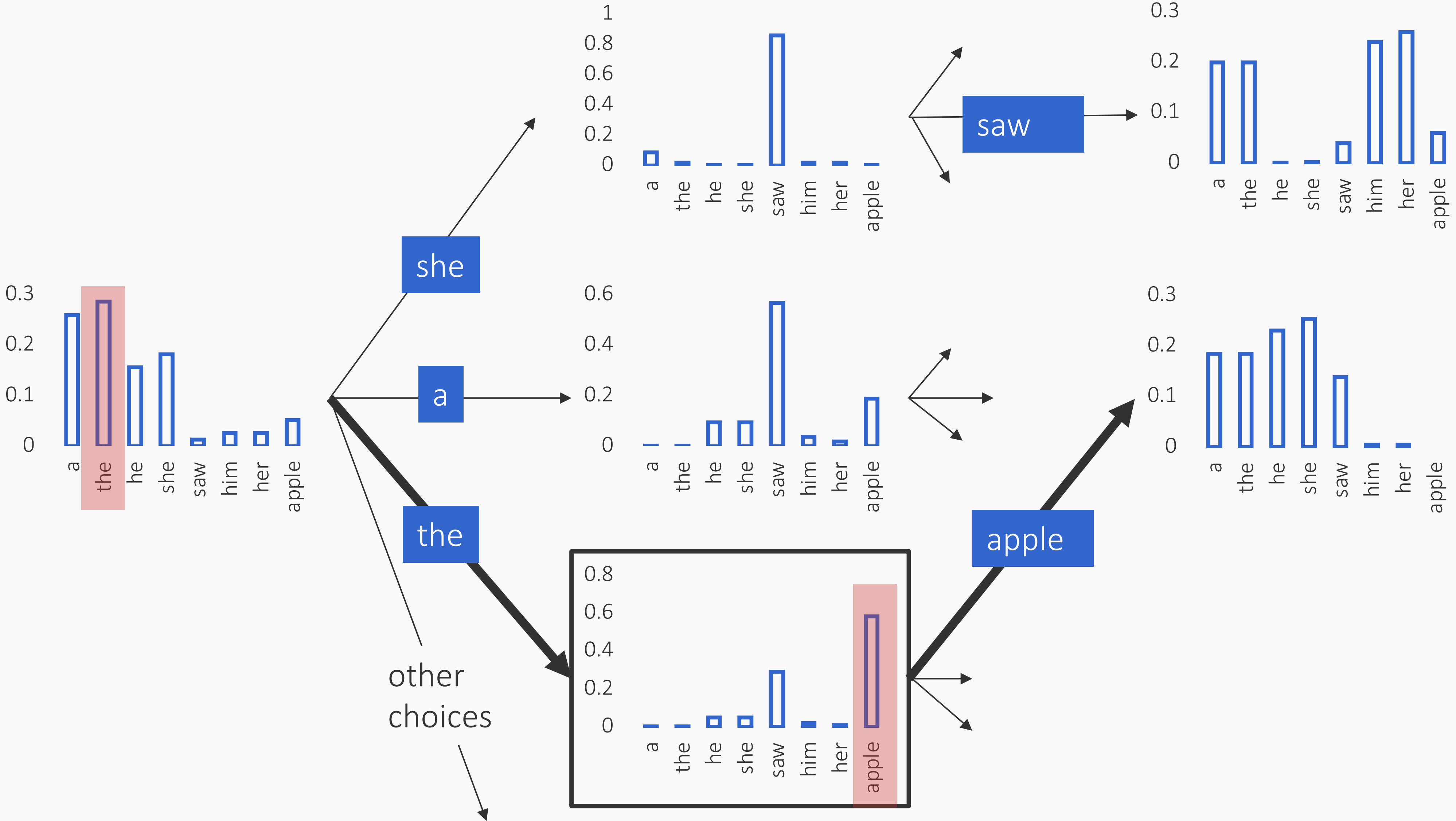

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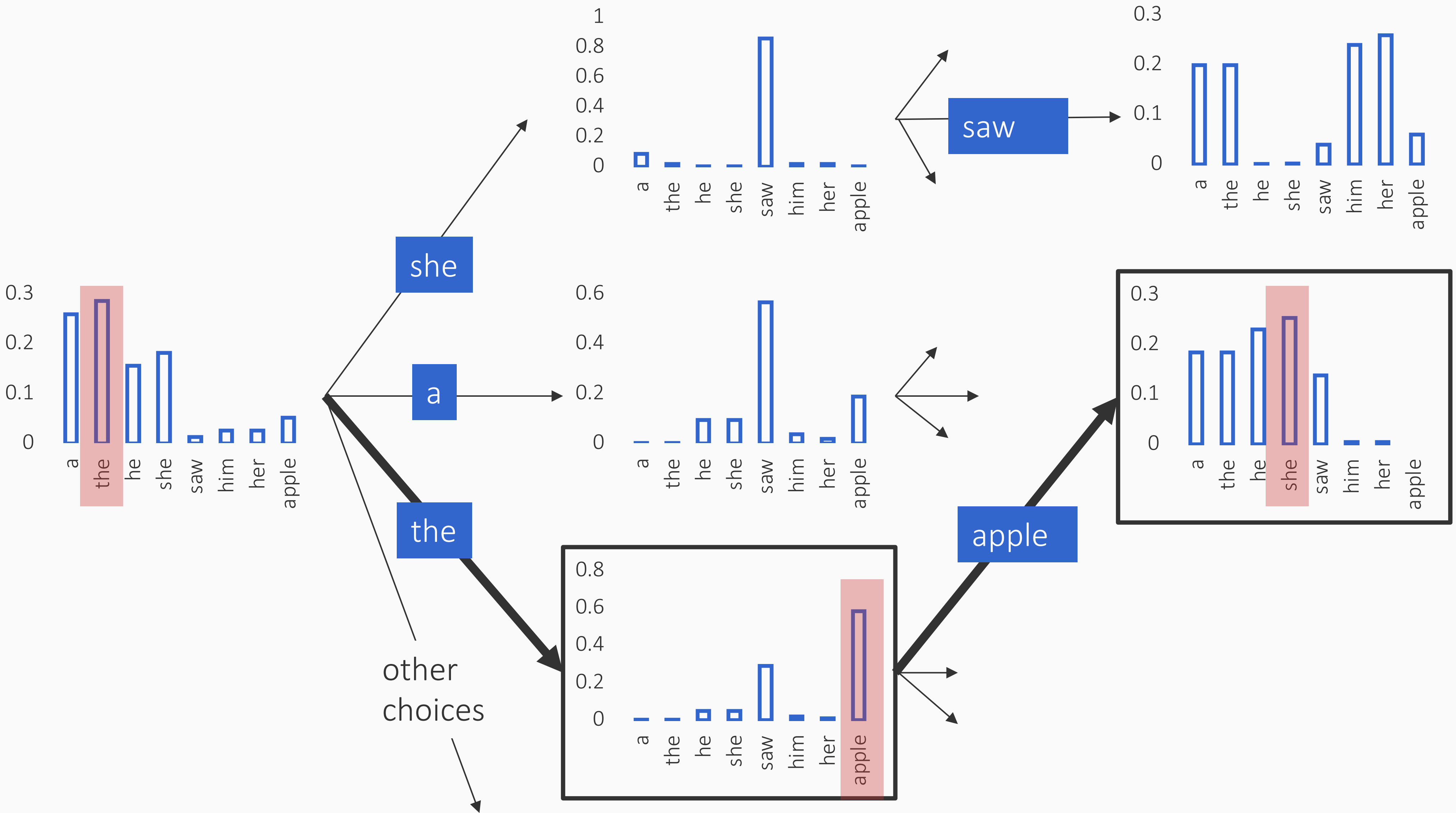

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Greedy decoding

At each step, pick the token that has the highest probability

$$w_n = \underset{\{v \in \text{Vocabulary}\}}{\operatorname{argmax}} P\left(v \mid w_0 w_1 \cdots w_{\{n-1\}} \right)$$

Pros:

- Simple
- Fast

Cons:

- If there is a high probability word later in the sequence, but to get there the model should have chosen a low probability word, this would get missed
- Also, in practice, the outputs can be repetitive

Different decoding strategies exist

Search based decoding

Deterministic approaches that involves searching the space of sequences to select one

- Greedy decoding
- Beam search

Sampling based decoding

Randomized approaches that involve sampling from the token conditional probability distribution

- Random sampling
- Top-K sampling
- Nucleus sampling

Beam search

- Keep size-limited priority queue of states
  - Called the beam, sorted by probability for the state
- At each step:
  - Explore all transitions from the current state
  - Add all to beam and trim the size

What we might really want to do is to explore the full search space.

We cannot. Beam search is a compromise

Beam search: A compromise

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Example: Suppose we have a beam of size k = 2

Beam search: A compromise

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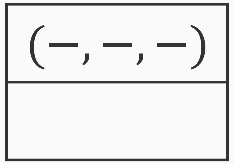

$(-, -, -)$

At the beginning, the beam has only one element, the start state

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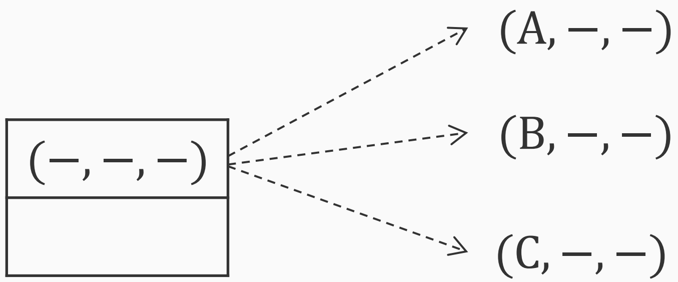

Expand all the states in the beam

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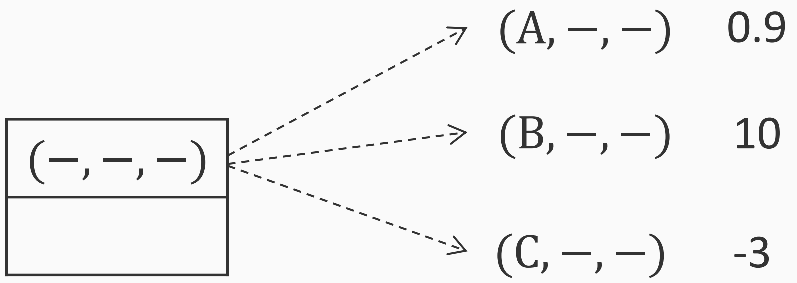

Expand all the states in the beam

Score the newly created states

Beam search: A compromise

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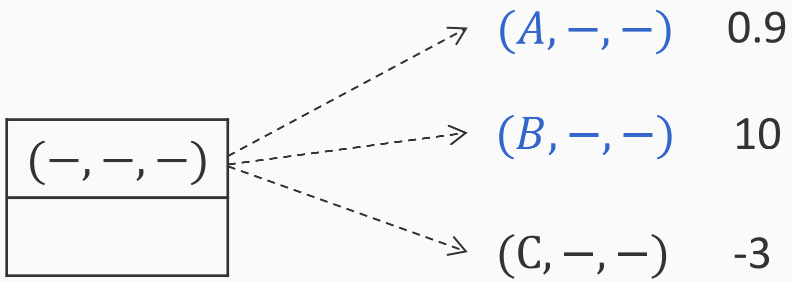

$(-,-,-)$

$(A,-,-)$ 0.9

$(B,-,-)$ 10

$(C,-,-)$ -3

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The top k new states form the new beam (sorted)

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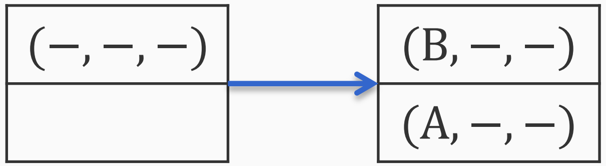

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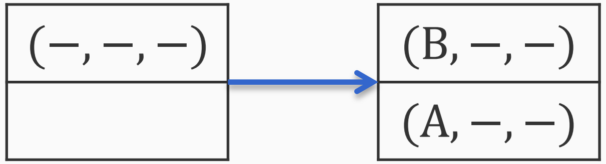

$(-, -, -)$

$(B, -, -)$

$(A, -, -)$

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Now we are ready for the next step

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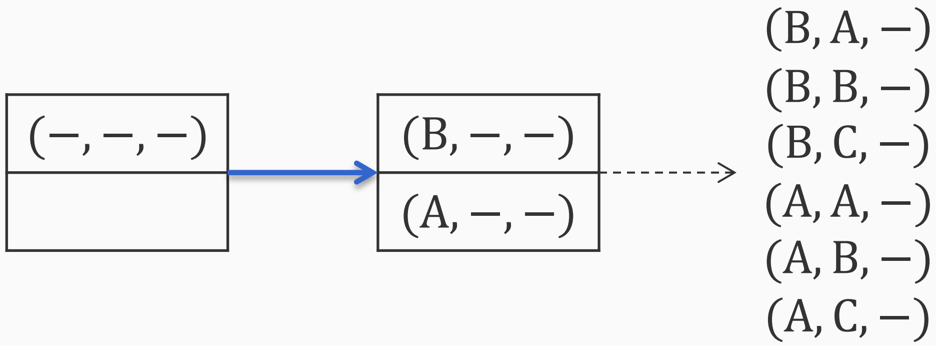

$(-,-,-)$

$(\mathrm{B},-,-)$
$(\mathrm{A},-,-)$

$(\mathrm{B},\mathrm{A},-)$
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$(\mathrm{B},\mathrm{C},-)$
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Expand all the states in the beam

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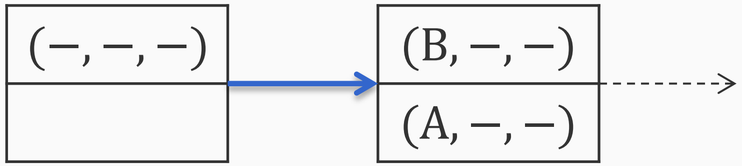

$(-,-,-)$

$(B,-,-)$

$(A,-,-)$

$(B, A, -)$	0.1
$(B, B, -)$	-3
$(B, C, -)$	10
$(A, A, -)$	20
$(A, B, -)$	-1
$(A, C, -)$	4.1

Expand all the states in the beam

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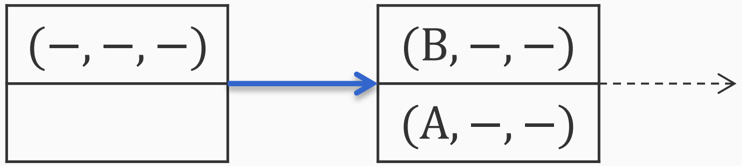

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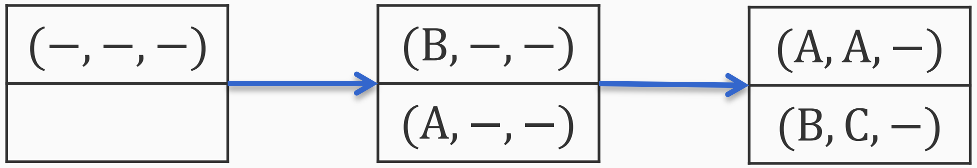

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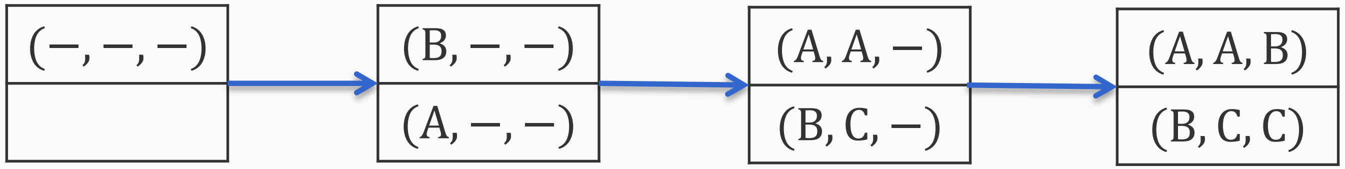

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	$(A,-,-)$	$(B,C,-)$	(B,C,C)

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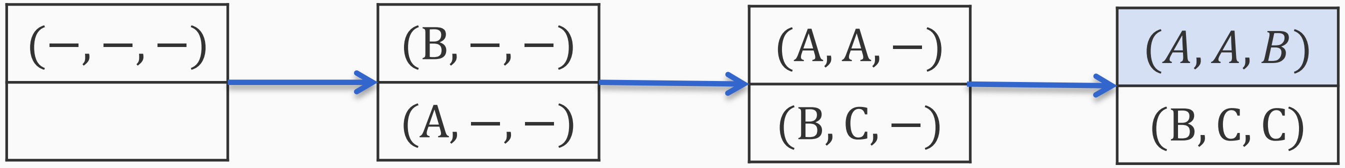

$(-,-,-)$

$(\mathrm{B},-,-)$ $(\mathrm{A},-,-)$

$(\mathrm{A},\mathrm{A},-)$ $(\mathrm{B},\mathrm{C},-)$

(A,A,B) $(\mathrm{B},\mathrm{C},\mathrm{C})$

Final answer: Top of the beam at the end of search

Beam Search

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Pros

- Explores more than greedy search. Greedy search is beam search with beam size 1
- In general, easy to implement, very popular
- We get a set of sequences that we can then re-order or use in other ways

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Cons

- A good state might fall out of the beam
- Can be still repetitive. Possible solution: add an n-gram penalty to penalize n-grams that get repeated
- Generated text may be *boring* for a reader

  Do we always choose the most probable next words? What makes a sequences of words interesting?

Different decoding strategies exist

Search based decoding

Deterministic approaches that involves searching the space of sequences to select one

- Greedy decoding
- Beam search

Sampling based decoding

Randomized approaches that involve sampling from the token conditional probability distribution

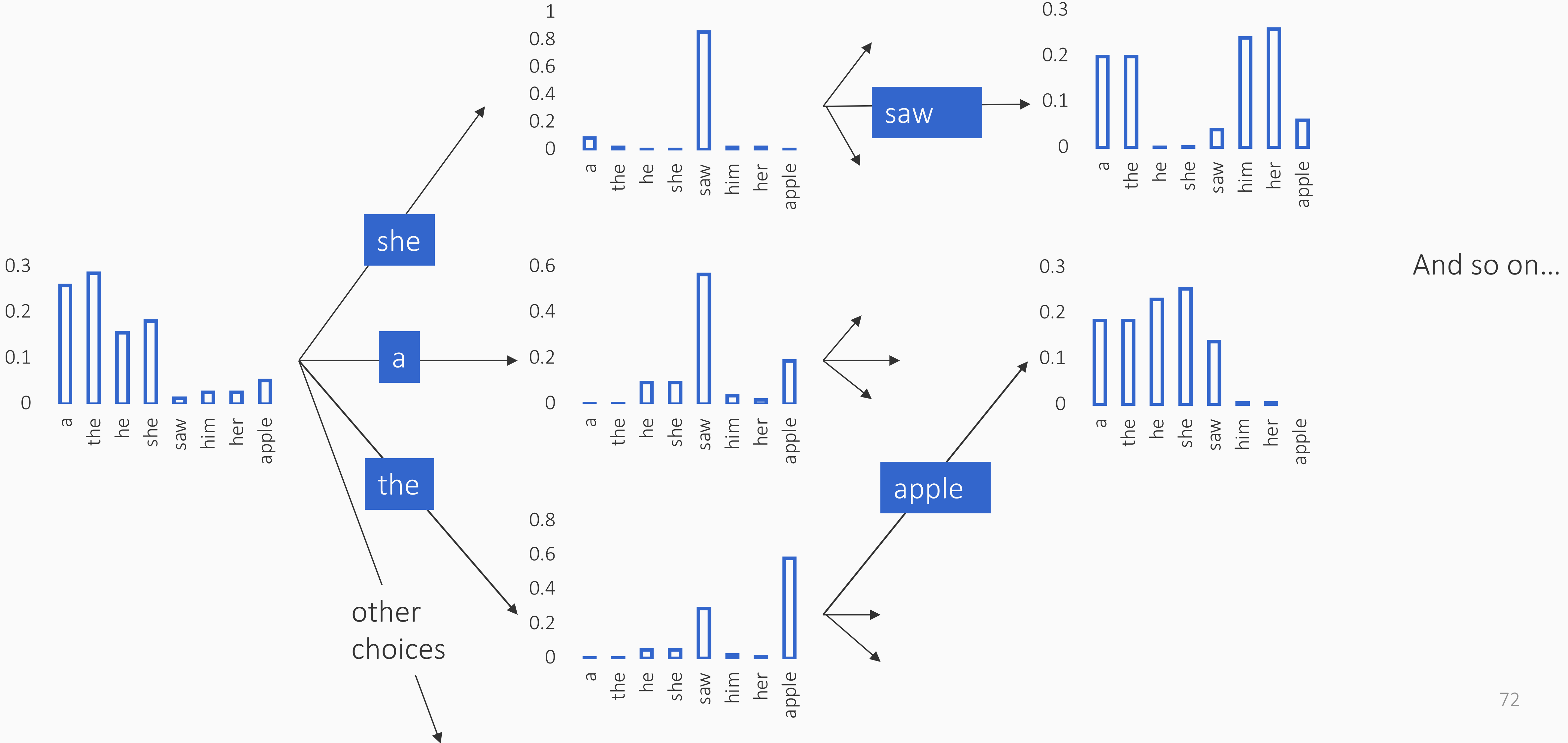
- Random sampling
- Top-K sampling
- Nucleus sampling

Sampling based approaches

Rather than picking the most probable next token, randomly pick one using the next token distribution

$$w_n \sim P\left(v \mid w_0 w_1 \cdots w_{\{n-1\}} \right)$$

Random sampling in our toy setting



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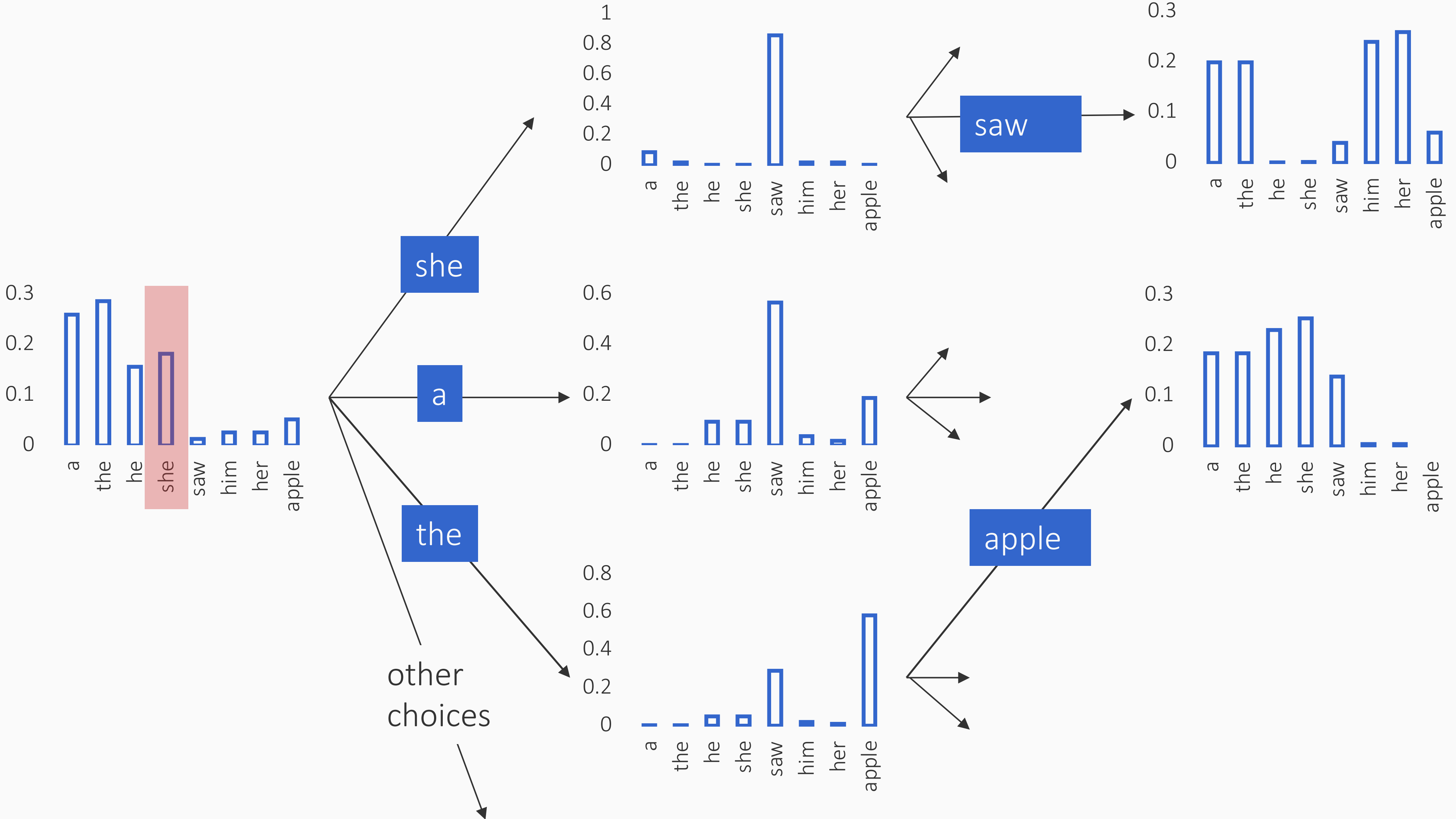

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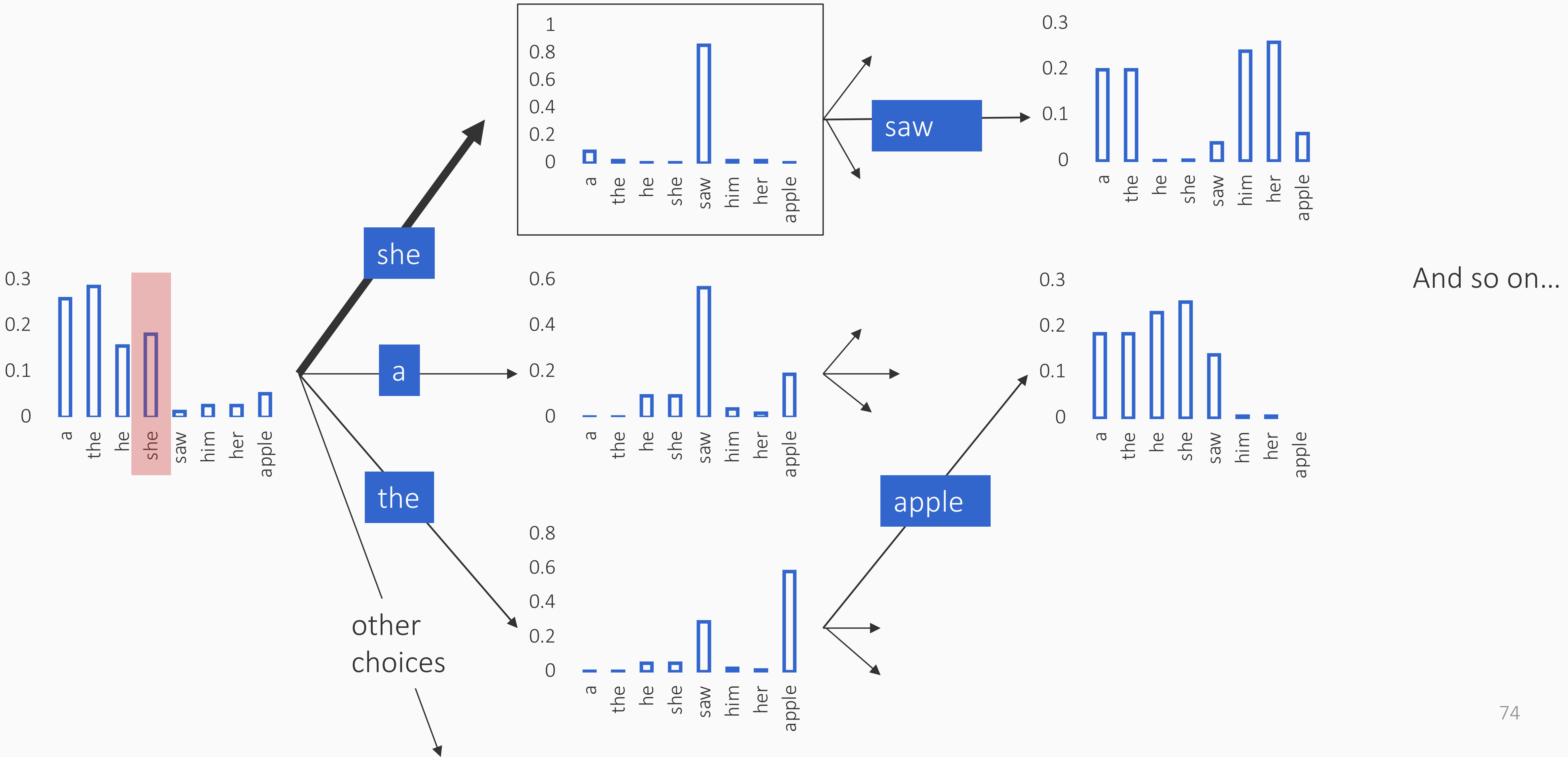

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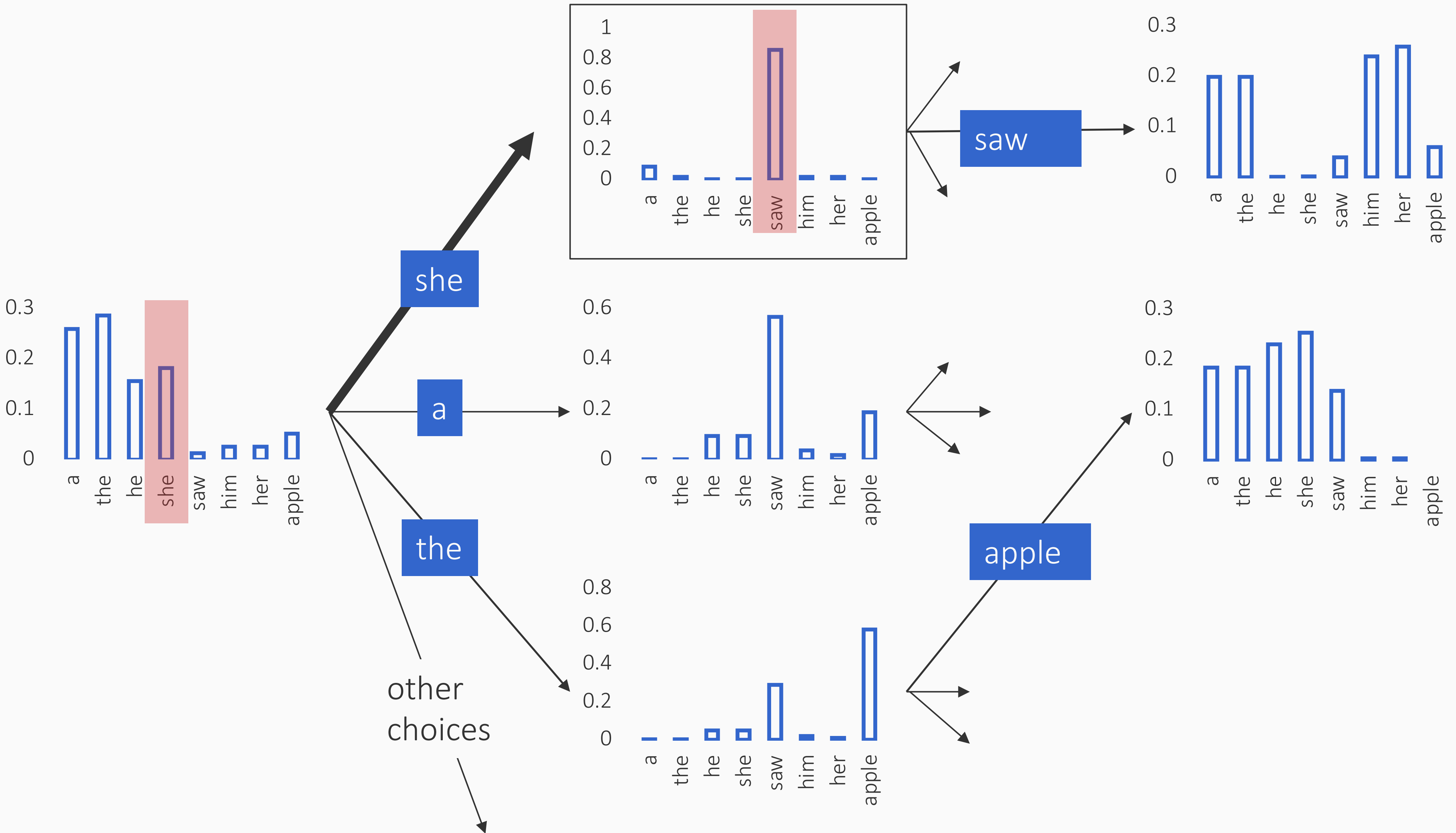

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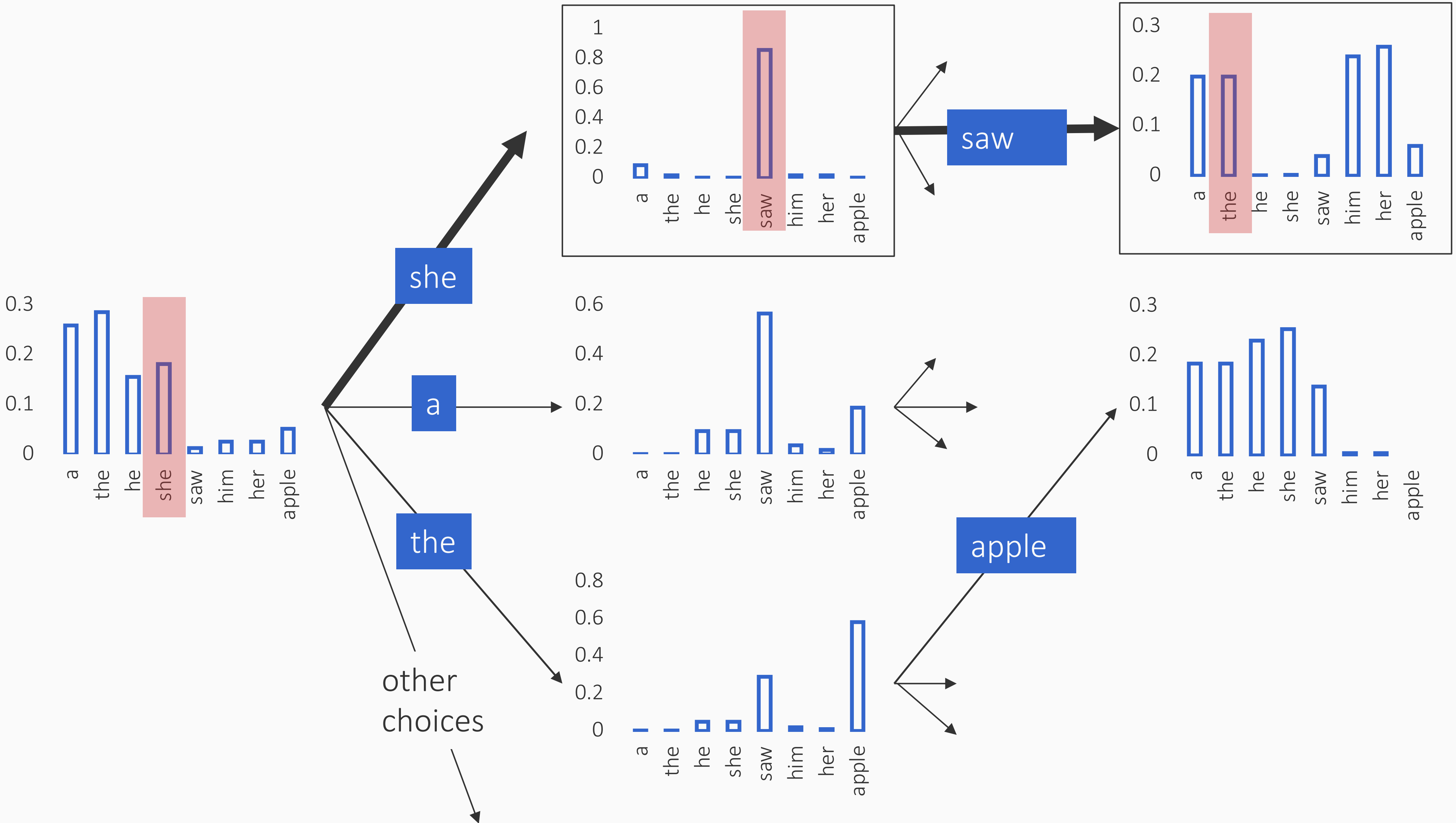

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Sampling based approaches

Rather than picking the most probable next token, randomly pick one using the next token distribution

$$w_n \sim P\left(v \mid w_0 w_1 \cdots w_{\{n-1\}} \right)$$

Pros

- Produces more interesting text
- Diverse outputs

Cons

- Does not produce coherent outputs. Why?

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$$P(token_i|context) = \frac{\exp\left(\frac{s_i}{T}\right)}{\sum_j \exp\left(\frac{s_j}{T}\right)}$$

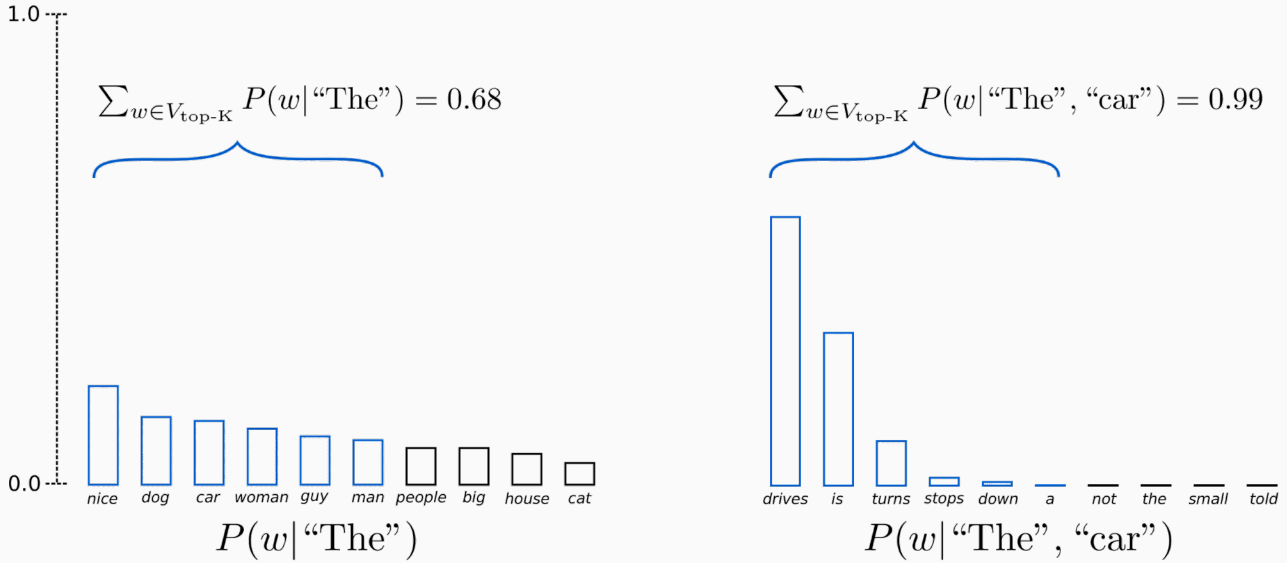
When T is lower than 1, probabilities get more sharp and lower probabilities get diminished

When T = 0, all the probability is placed on the token with the highest s_i → Greedy decoding

Other variants of sampling

Top-K sampling

- Instead of sampling from all the words, pick the K most probable words, and distribute the probability mass among them
- GPT2 used this



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Nucleus sampling (sometimes called Top-p sampling)

- Instead of choosing a fixed K, choose as many words as necessary so that the total probability allocated to them is at least p

Both methods seem to produce more fluent text than greedy and beam search

This lecture: Decoding algorithms

How to predict a sequence

- The setup: An abstract auto-regressive model
- Decoding algorithms
  - Greedy decoding
  - Beam search
  - Random sampling
  - Top-K sampling
  - Nucleus sampling