#### Recurrent Neural Networks



#### Overview

- 1. Modeling sequences
- 2. Recurrent neural networks: An abstraction
- 3. Usage patterns for RNNs
- 4. BiDirectional RNNs
- 5. A concrete example: The Elman RNN
- 6. The vanishing gradient problem
- 7. Long short-term memory units

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- 1. Modeling sequences
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Salt Lake City

Words are sequences of characters

Salt Lake City

John lives in Salt Lake City

Sentences are sequences of words

```
Salt Lake City

John lives in Salt Lake City
```

John lives in Salt Lake City. He enjoys hiking with his dog. His cat hates hiking.

Paragraphs are sequences of sentences

Salt Lake City

John lives in Salt Lake City

John lives in Salt Lake City. He enjoys hiking with his dog. His cat hates hiking.

And so on... inputs are naturally sequences at different levels

```
Salt Lake City
```

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Outputs can also be sequences

```
Salt Lake City

John lives in Salt Lake City

John lives in Salt Lake City. He enjoys hiking with his dog. His cat hates hiking.
```

John lives in Salt Lake City

Part-of-speech tags form a sequence

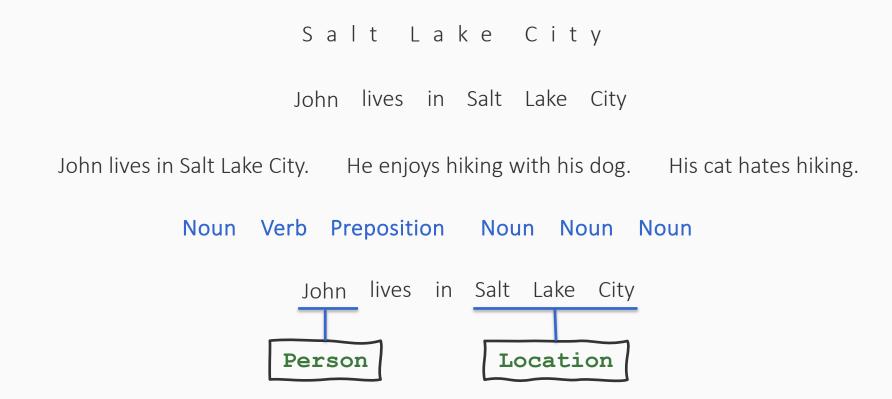
```
Salt Lake City

John lives in Salt Lake City
```

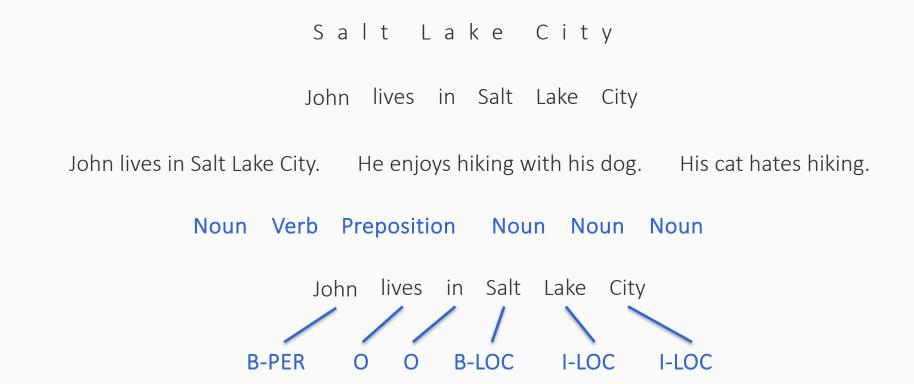
John lives in Salt Lake City. He enjoys hiking with his dog. His cat hates hiking.



Part-of-speech tags form a sequence



Even things that don't look like a sequence can be made to look like one Example: Named entity tags



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```
John lives in Salt Lake City

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John lives in Salt Lake City. He enjoys hiking with his dog. His cat hates hiking.

Noun Verb Preposition Noun Noun Noun

B-PER O O B-LOC I-LOC I-LOC
```

And we can get very creative with such encodings

Example: We can encode parse trees as a sequence of decisions needed to construct the tree

Salt Lake City

John lives in Salt Lake City

Natural question: How do we model sequential inputs and outputs?

JUHIT HVES HI SAIT LAKE CITY. THE CHIJOYS HIKHIR WITH HIS GOR. THIS CAT HATES HIKHIR.

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Example: We can encode parse trees as a sequence of decisions needed to construct the tree

```
Salt Lake City

John lives in Salt Lake City
```

Natural question: How do we model sequential inputs and outputs?

More concretely, we need a mechanism that allows us to

- 1. Capture sequential dependencies between inputs
- 2. Model uncertainty over sequential outputs

And we can get very creative with such encodings

Example: We can encode parse trees as a sequence of decisions needed to construct the tree

#### Modeling sequences: The problem

Suppose we want to build a language model that computes the probability of sentences

We can write the probability as

$$P(x_1, x_2, x_3, \dots, x_n) = \prod_i P(x_i \mid x_1, x_2, \dots, x_{i-1})$$

It was a bright cold day in April.

P(It was a bright cold day in April) =

$$P(\text{It was a bright cold day in April}) = \\ P(\text{It}) \times \longleftarrow$$
 Probability of a word starting a sentence

```
P(\mathrm{It\ was\ a\ bright\ cold\ day\ in\ April}) =
P(\mathrm{It}) \times \hspace{1cm} \text{Probability\ of\ a\ word\ starting\ a\ sentence}
P(\mathrm{was}|\mathrm{It}) \times \hspace{1cm} \text{Probability\ of\ a\ word\ following\ "It"}
P(\mathrm{a}|\mathrm{It\ was}) \times \hspace{1cm} \text{Probability\ of\ a\ word\ following\ "It\ was"}
```

```
P(\text{It was a bright cold day in April}) = \\ P(\text{It}) \times \longleftarrow \text{Probability of a word starting a sentence} \\ P(\text{was}|\text{It}) \times \longleftarrow \text{Probability of a word following "It"} \\ P(\text{a}|\text{It was}) \times \longleftarrow \text{Probability of a word following "It was"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probability of a word following "It was a"} \\ P(\text{bright}|\text{It was a}) \times \longleftarrow \text{Probabil
```

```
P(\mathrm{It} \ \mathrm{was} \ \mathrm{a} \ \mathrm{bright} \ \mathrm{cold} \ \mathrm{day} \ \mathrm{in} \ \mathrm{April}) =
P(\mathrm{It}) \times \longleftarrow \mathrm{Probability} \ \mathrm{of} \ \mathrm{a} \ \mathrm{word} \ \mathrm{starting} \ \mathrm{a} \ \mathrm{sentence}
P(\mathrm{was}|\mathrm{It}) \times \longleftarrow \mathrm{Probability} \ \mathrm{of} \ \mathrm{a} \ \mathrm{word} \ \mathrm{following} \ \mathrm{``lt''}
P(\mathrm{a}|\mathrm{It} \ \mathrm{was}) \times \longleftarrow \mathrm{Probability} \ \mathrm{of} \ \mathrm{a} \ \mathrm{word} \ \mathrm{following} \ \mathrm{``lt} \ \mathrm{was} \ \mathrm{a'}
P(\mathrm{bright}|\mathrm{It} \ \mathrm{was} \ \mathrm{a}) \times \longleftarrow \mathrm{Probability} \ \mathrm{of} \ \mathrm{a} \ \mathrm{word} \ \mathrm{following} \ \mathrm{``lt} \ \mathrm{was} \ \mathrm{a''}
P(\mathrm{cold}|\mathrm{It} \ \mathrm{was} \ \mathrm{a} \ \mathrm{bright}) \times
P(\mathrm{day}|\mathrm{It} \ \mathrm{was} \ \mathrm{a} \ \mathrm{bright} \ \mathrm{cold}) \times \cdots
```

## A history-based model

$$P(x_1, x_2, \dots, x_n) = \prod_{i=1}^n P(x_i | x_1, x_2, \dots, x_{i-1})$$

Each token is dependent on all the tokens that came before it

- Simple conditioning
- Each  $P(x_i \mid ...)$  is a multinomial probability distribution over the tokens

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#### What is the problem here?

- How many parameters do we have?
  - Grows with the size of the sequence!

#### The traditional solution: Lose the history

Make a modeling assumption

Example: The first order Markov model assumes that

$$P(x_i \mid x_1, x_2, \dots, x_{i-1}) = P(x_i \mid x_{i-1})$$

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This allows us to simplify

$$P(x_1, x_2, x_3, \cdots, x_n) = \prod_i P(x_i \mid x_1, x_2, \cdots, x_{i-1})$$
These dependencies are ignored

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# Example: Another language model

```
P(\text{It was a bright cold day in April}) = \\ P(\text{It}) \times & \qquad \qquad \text{Probability of a word starting a sentence} \\ P(\text{was}|\text{It}) \times & \qquad \qquad \text{Probability of a word following "It"} \\ P(\text{a}|\text{was}) \times & \qquad \qquad \text{Probability of a word following "was"} \\ P(\text{bright}|\text{a}) \times & \qquad \qquad \text{Probability of a word following "a"} \\ P(\text{cold}|\text{bright}) \times & \qquad \qquad P(\text{day}|\text{cold}) \times \cdots
```

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#### It was a bright cold day in April

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If there are K tokens/states, how many parameters do we need?

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```

If there are K tokens/states, how many parameters do we need?  $O(K^2)$ 

#### Can we do better?

• Can we capture the meaning of the entire history without arbitrarily growing the number of parameters?

Or equivalently, can we discard the Markov assumption?

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• Can we capture the meaning of the entire history without arbitrarily growing the number of parameters?

Or equivalently, can we discard the Markov assumption?

- Can we represent arbitrarily long sequences as fixed sized vectors?
  - Perhaps to provide features for subsequent classification
- Answer: Recurrent neural networks (RNNs)