Recurrent Neural Networks

THE UNIVERSITY OF UTAH

Overview

1. Modeling sequences
2. Recurrent neural networks: An abstraction
3. Usage patterns for RNNs
4. BiDirectional RNNs
5. A concrete example: The Elman RNN
6. The vanishing gradient problem
7. Long short-term memory units

Overview

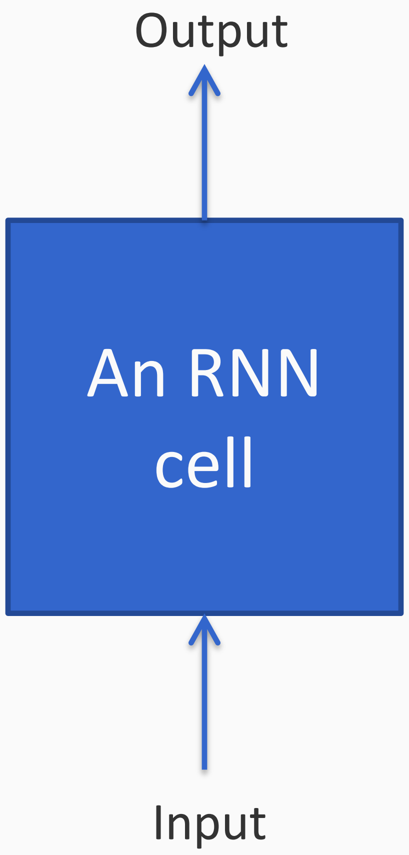
Recurrent neural networks

- First introduced by Elman 1990
- Provides a mechanism for representing sequences of arbitrary length into vectors that encode the sequential information
- A useful design abstraction if you’d like to work with sequential data
  - Till transformers came along, for a few years, RNNs were the best tools for representing text sequences

The RNN abstraction

A high level overview that doesn't go into details

An RNN cell is a unit of differentiable compute that maps inputs to outputs



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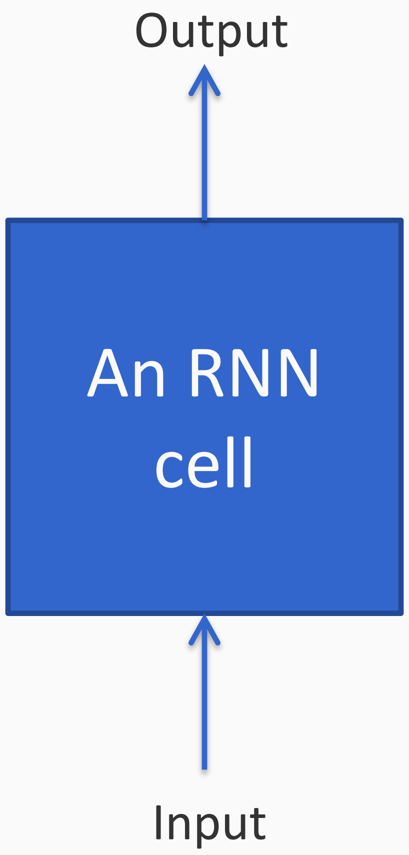

So far, no way to build a sequence of such cells

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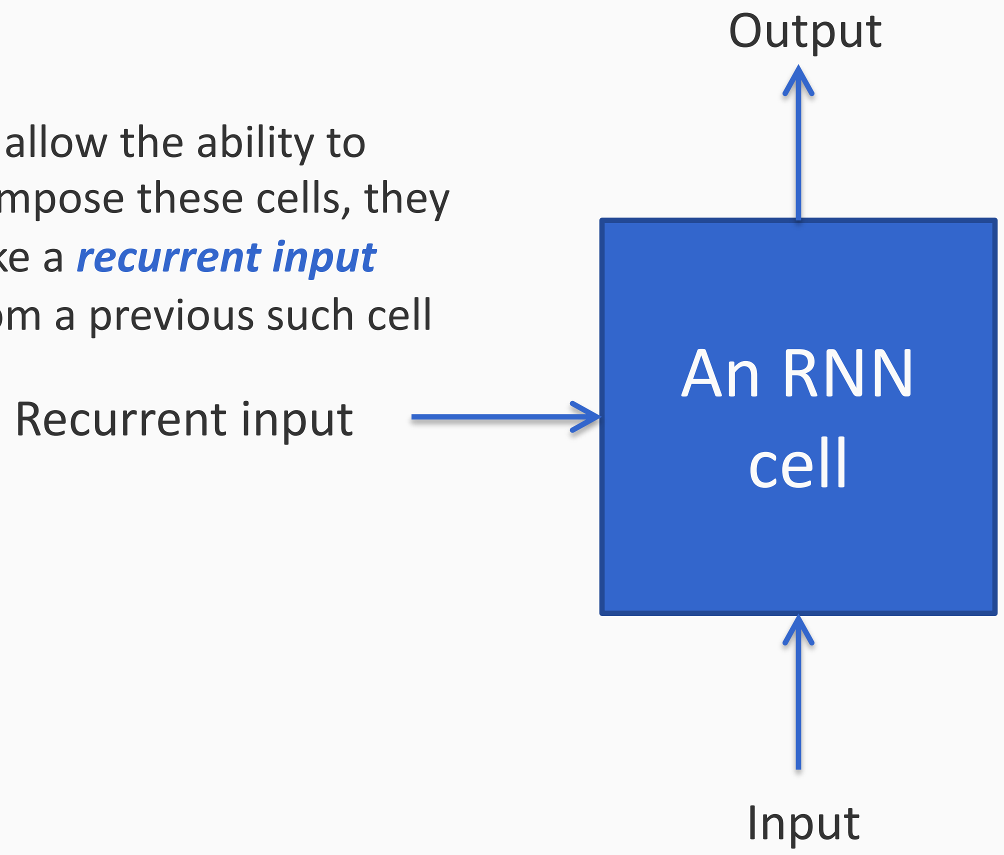
To allow the ability to compose these cells, they take a ***recurrent input*** from a previous such cell



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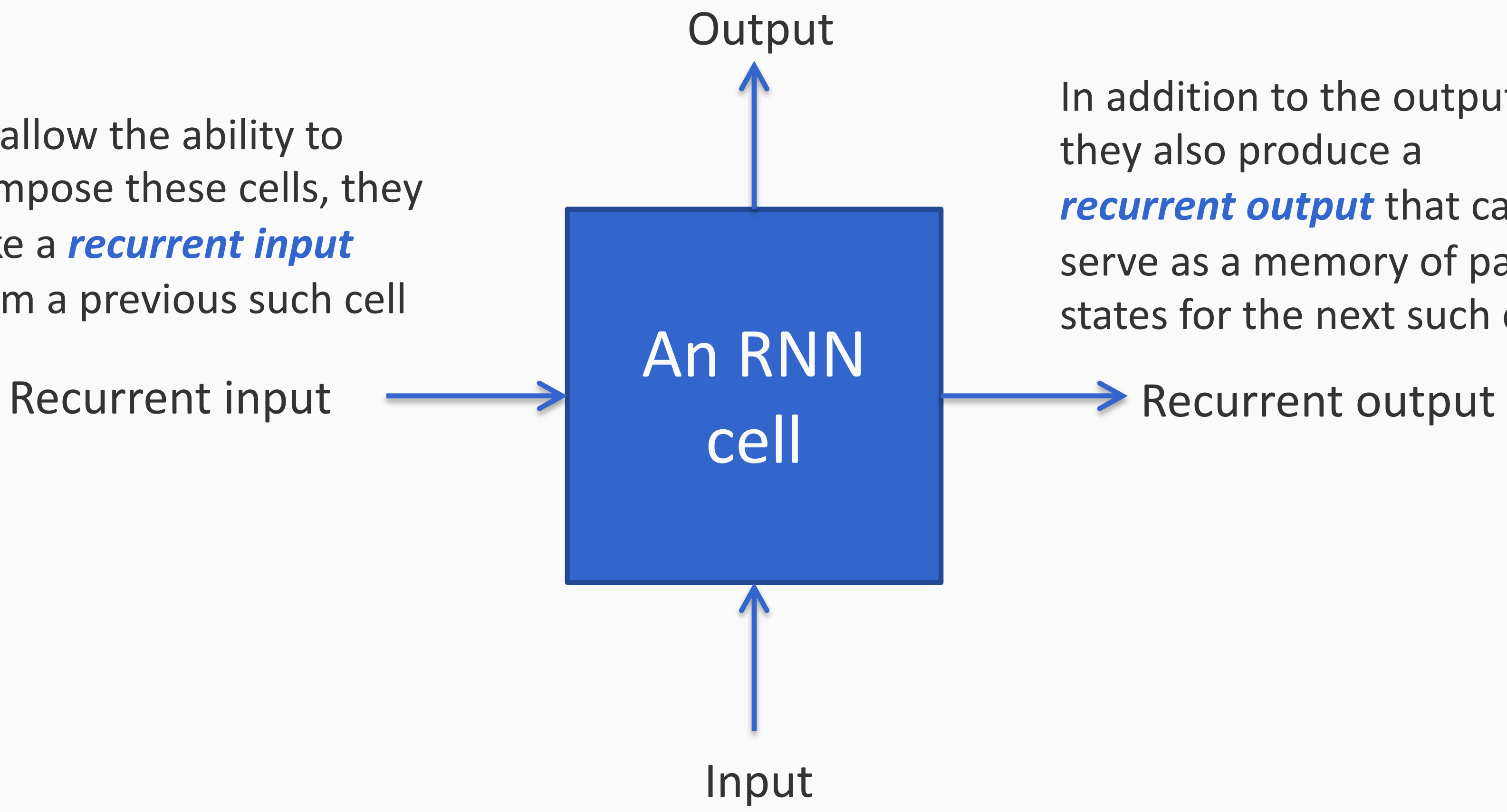

In addition to the output, they also produce a ***recurrent output*** that can serve as a memory of past states for the next such cell

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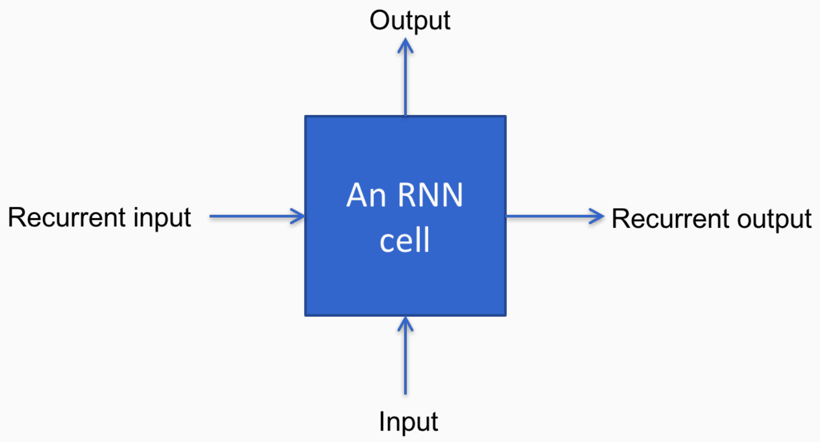


Conceptually two operations

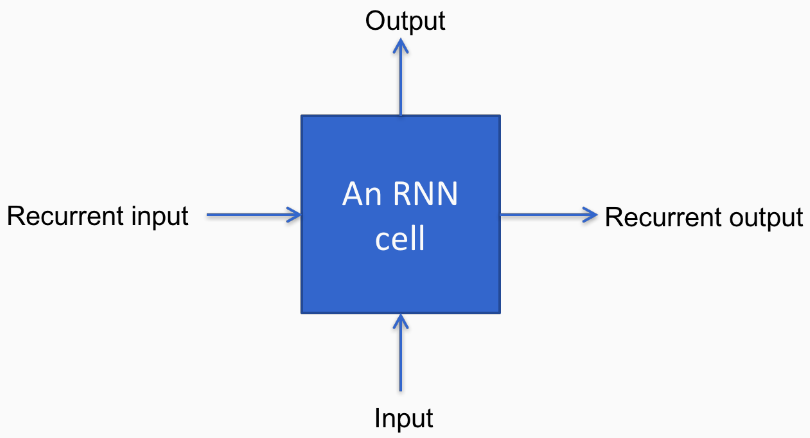
Using the input and the recurrent input (also called the previous cell state), compute

1. The next cell state

2. The output

The RNN abstraction: A simple example

John lives in Salt Lake City



This template is **unrolled** for each input

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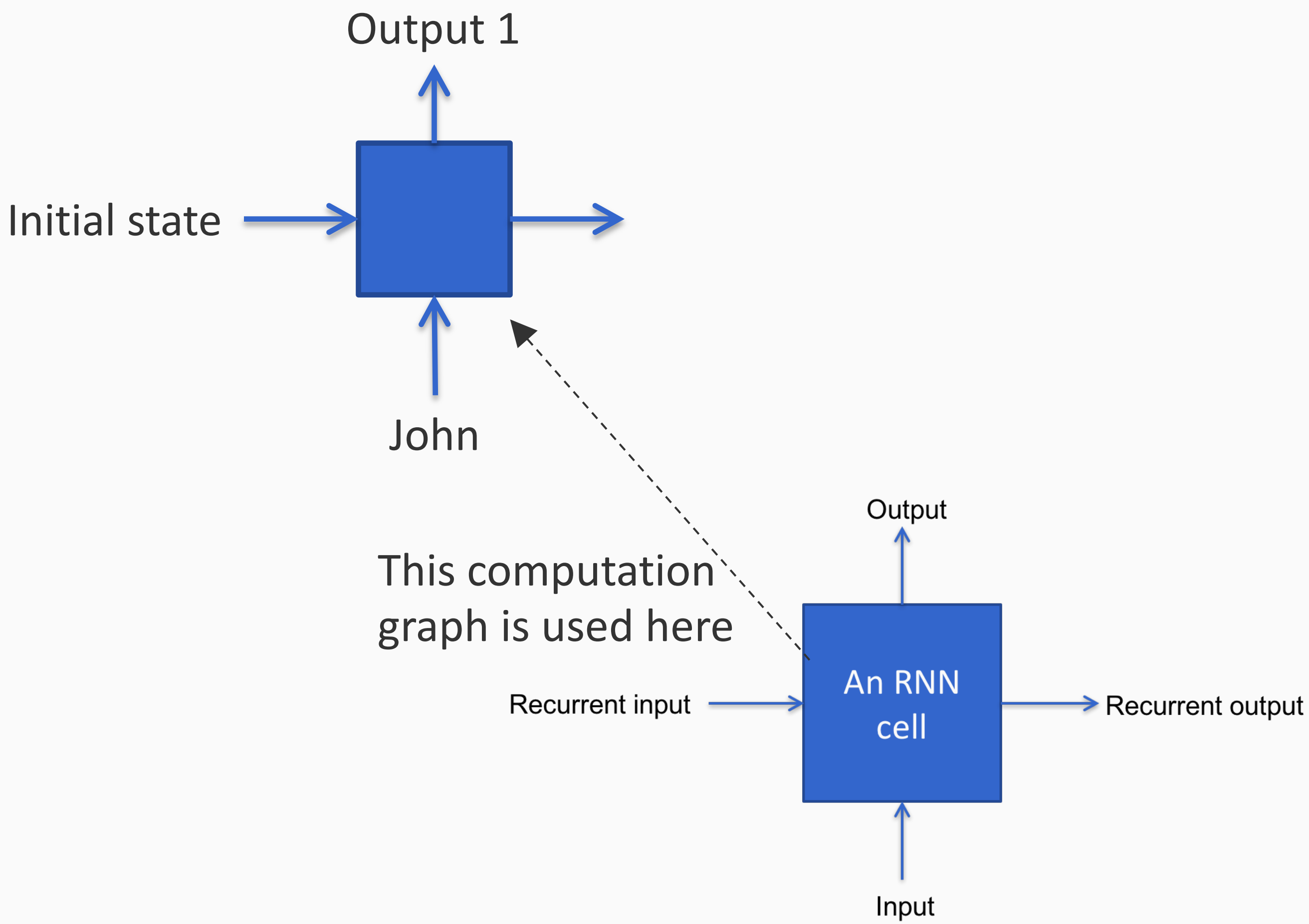

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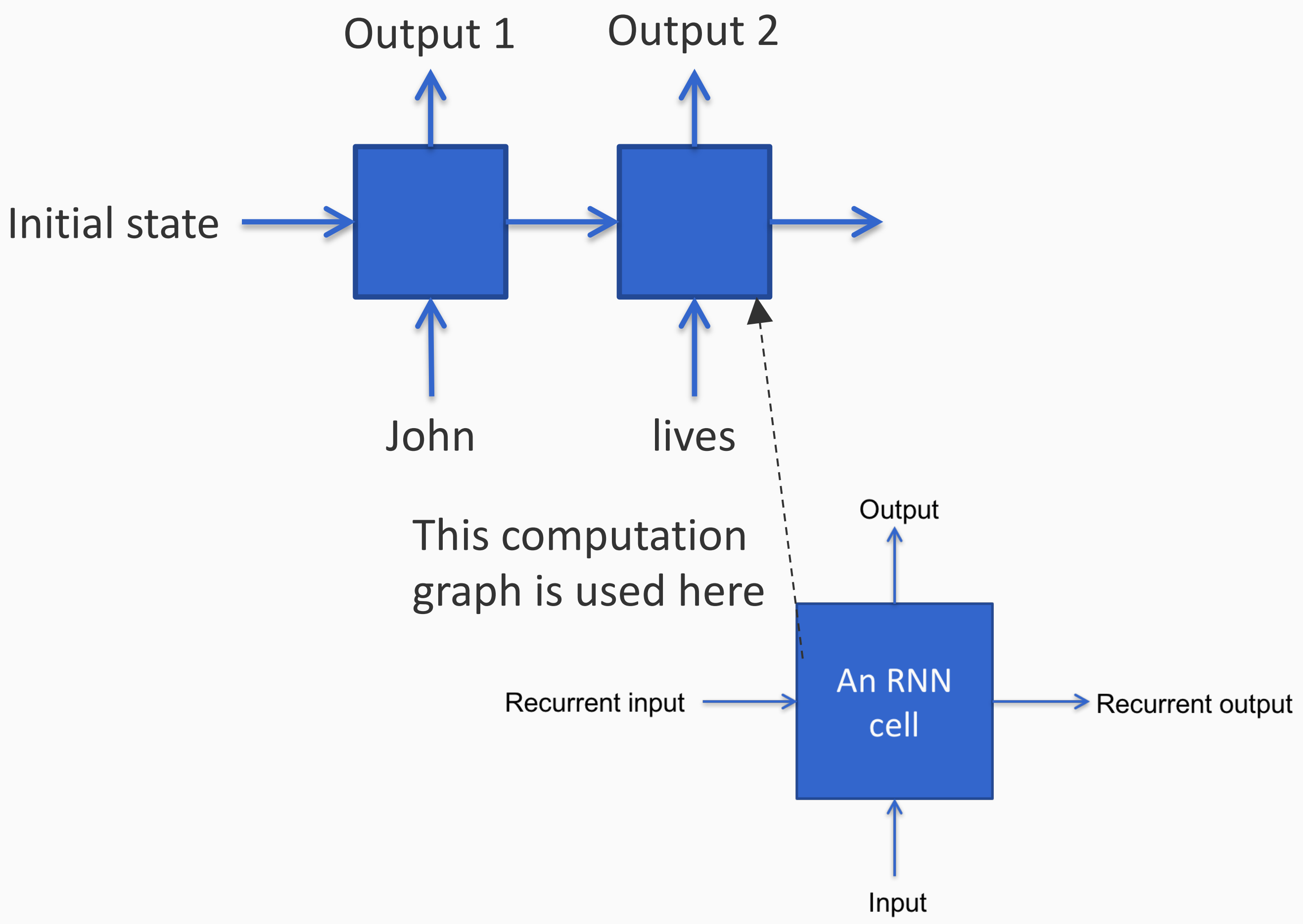

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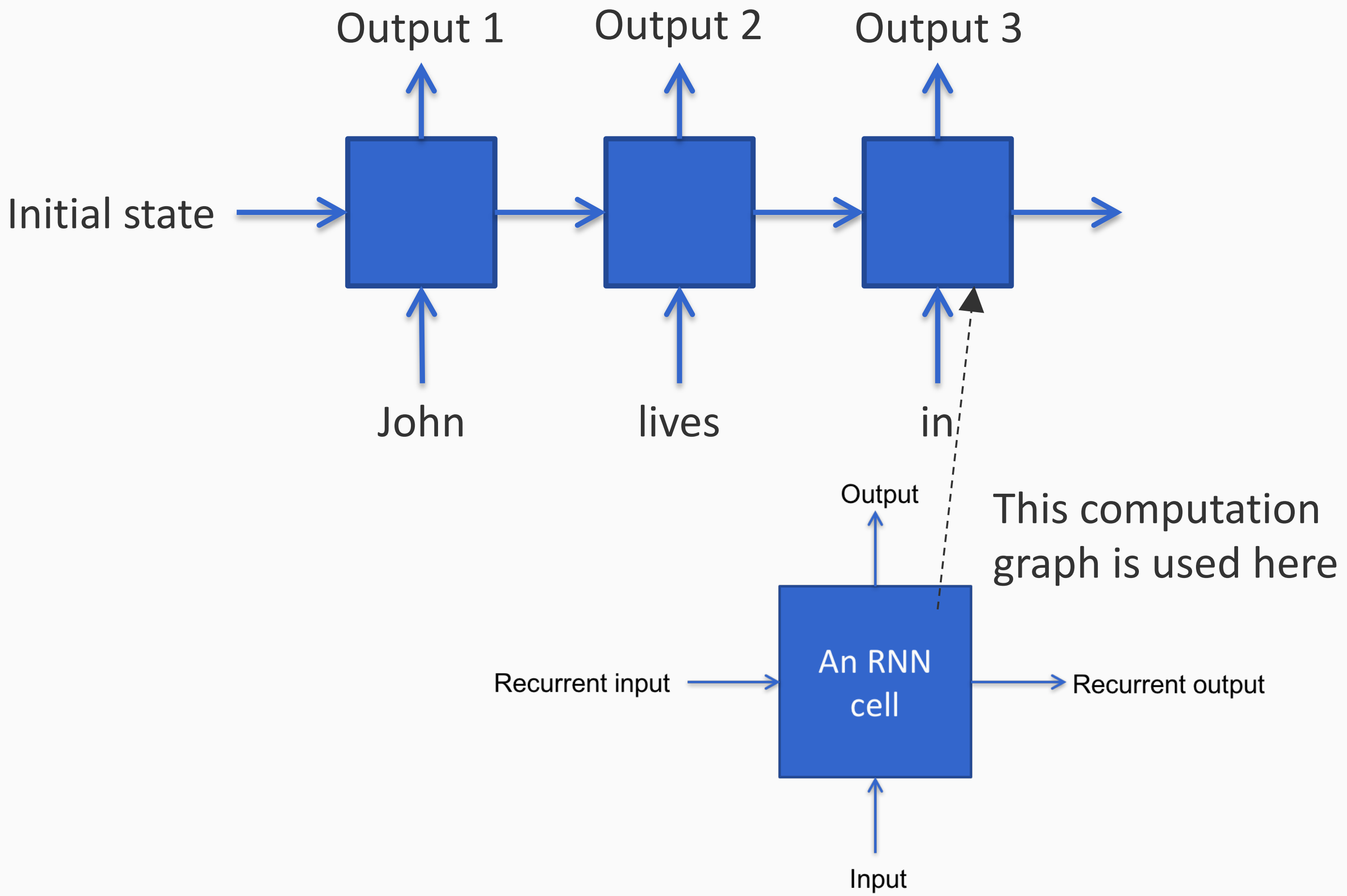

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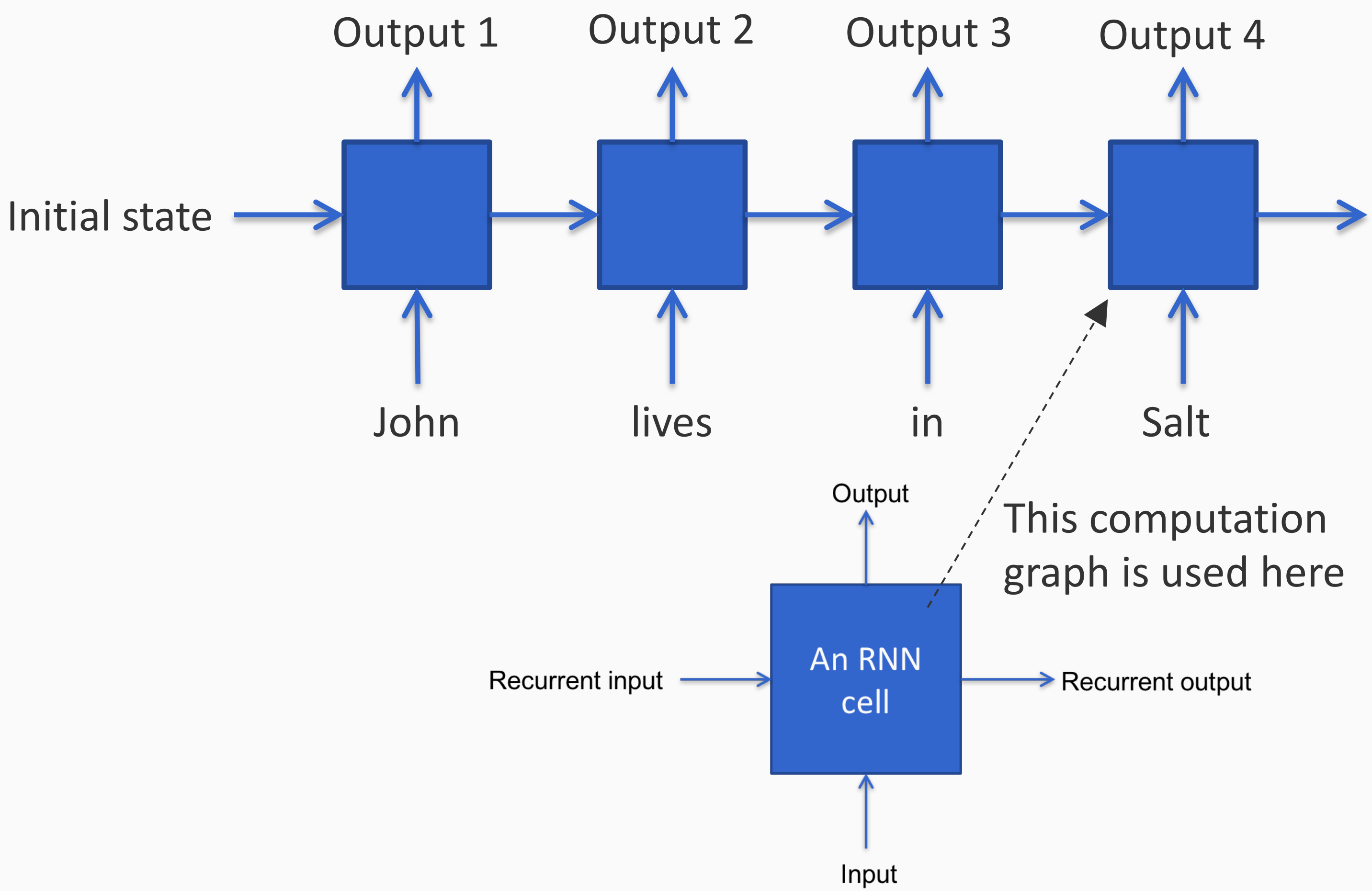

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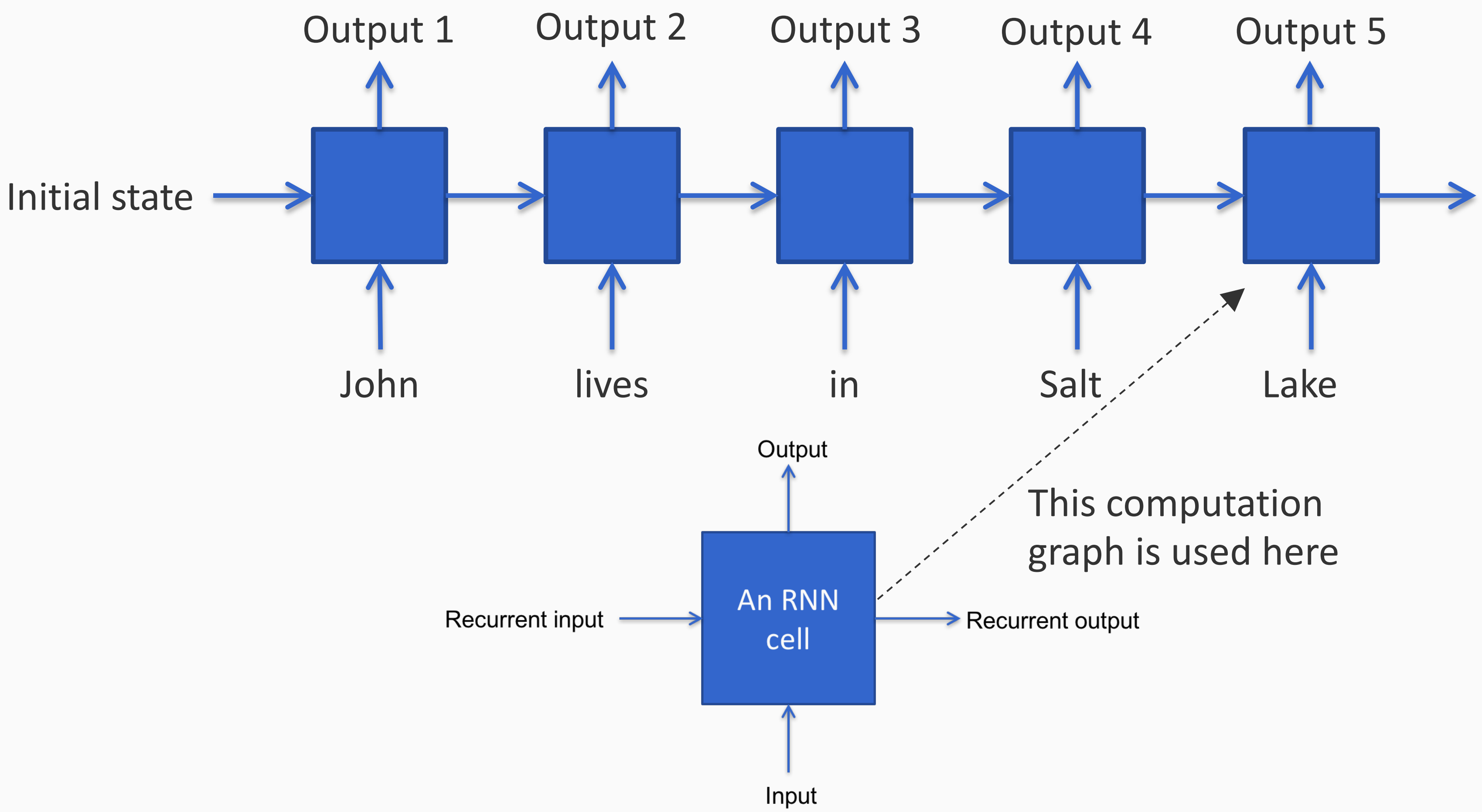

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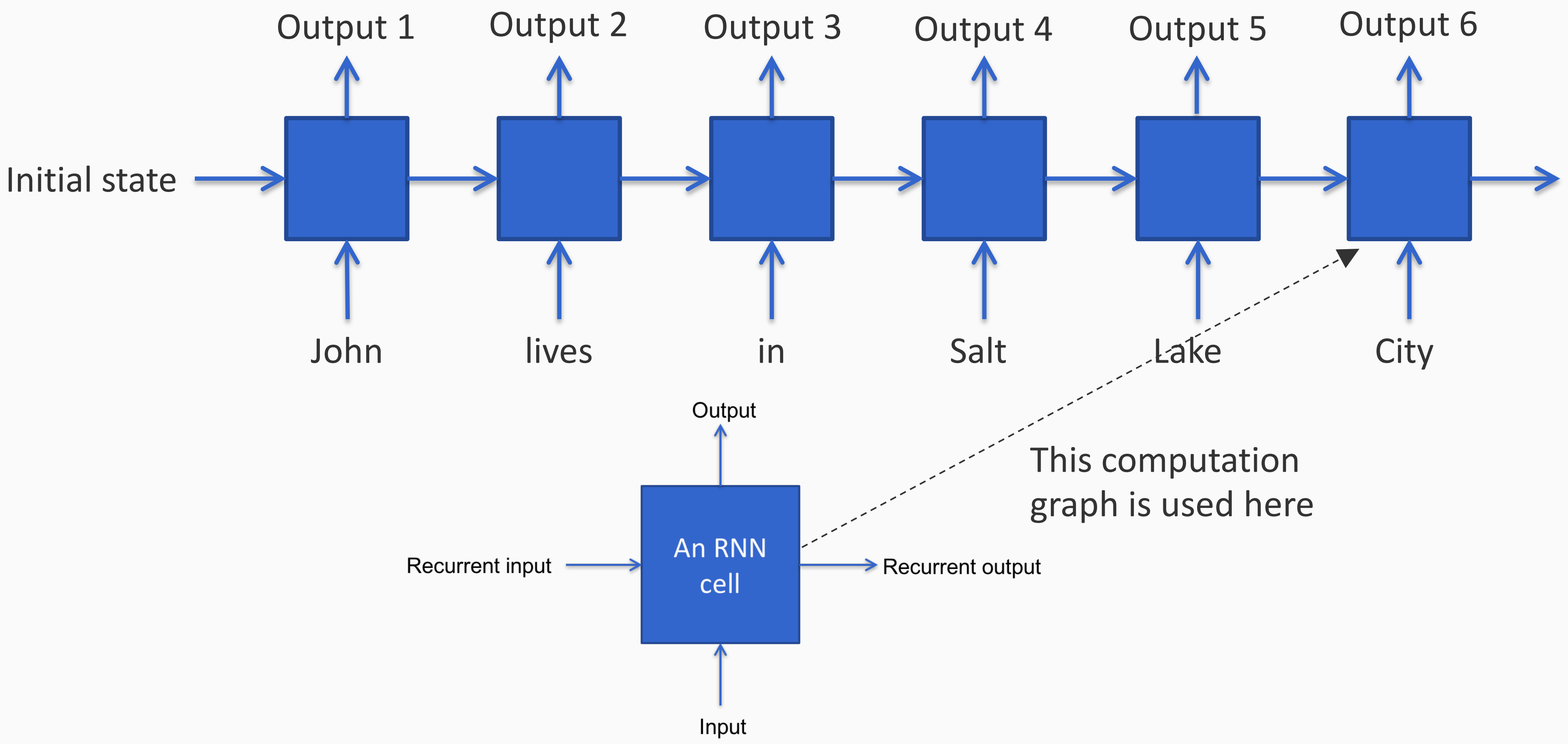
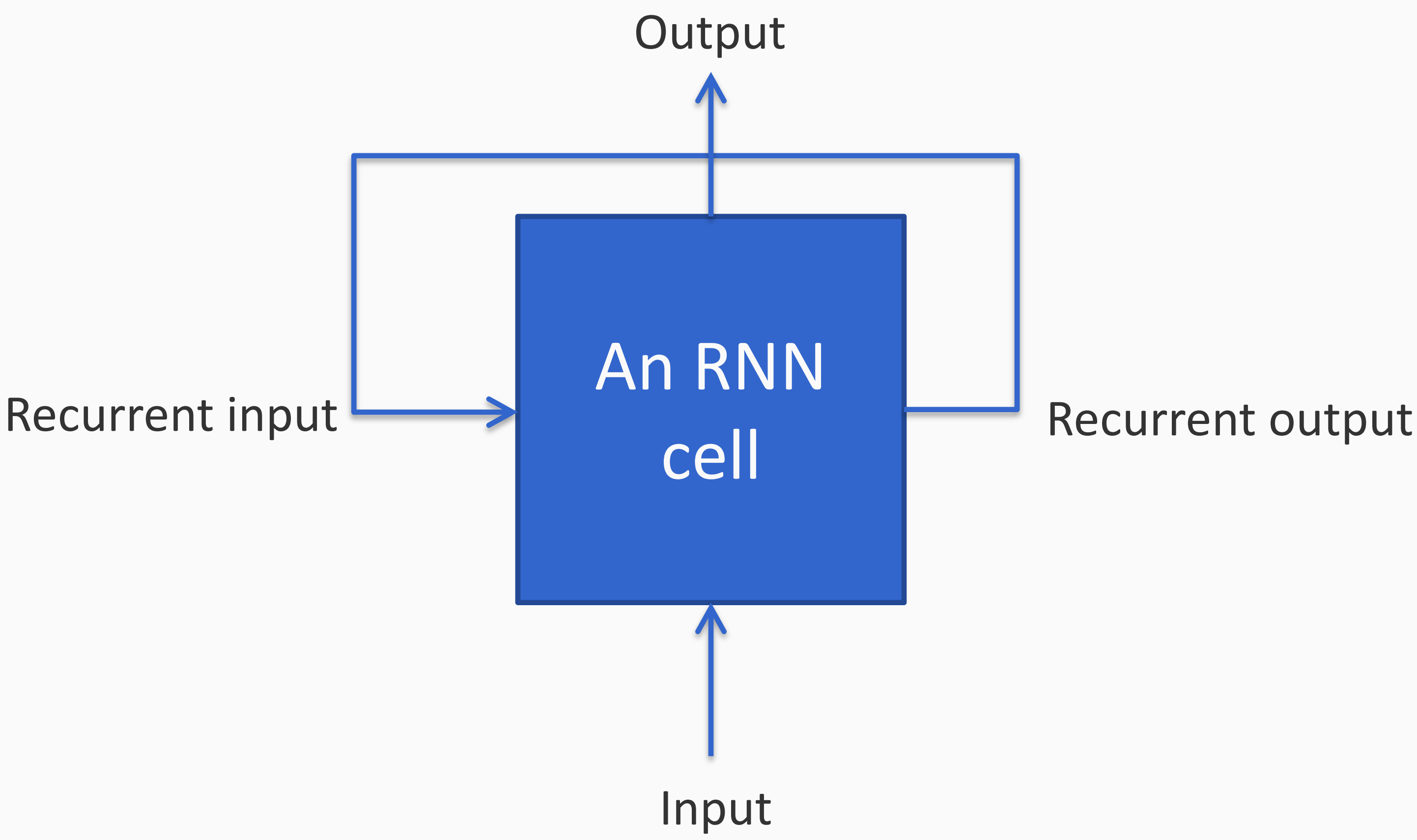


The RNN abstraction

Sometimes this is represented as a "neural network with a loop".

But really, when unrolled, there are no loops. Just a big feedforward network.



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Both these functions can be parameterized. That is, they can be neural networks whose parameters are trained.

What does unrolling the RNN do?

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Encodes the sequence till t=2 into a single vector

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  - $\mathbf{s}_3 = \mathrm{R}(\mathbf{s}_2, \mathbf{x}_3) = \mathrm{R}(\mathrm{R}(\mathrm{R}(\mathbf{s}_0, \mathbf{x}_1), \mathbf{x}_2), \mathbf{x}_3)$

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What does unrolling the RNN do?

- At each step:
  - Compute the next cell state: $\mathbf{s}_t = \mathrm{R}(\mathbf{s}_{t-1}, \mathbf{x}_t)$
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- We can write this as:
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Encodes the sequence till t=4 into a single vector

What does unrolling the RNN do?

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  - $\mathbf{s}_4 = \mathrm{R}(\mathbf{s}_3, \mathbf{x}_4) = \mathrm{R}(\mathrm{R}(\mathrm{R}(\mathbf{s}_0, \mathbf{x}_1), \mathbf{x}_2), \mathbf{x}_3), \mathbf{x}_4)$

  … and so on